

# Multinational Firms, Trade, and the Trade-Comovement Puzzle

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April 21, 2018

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## Abstract

Existing empirical studies show a strong positive correlation between bilateral trade and business cycle comovement within country-pairs. I show that for OECD economies, this relationship weakens considerably when bilateral FDI stock is controlled for, while FDI is significant. I develop a two-country business cycle model with heterogeneous firms, international trade, and multinational activity to explain this empirical finding. The calibrated model generates the positive relationship between trade and comovement, and between FDI and comovement. In addition, the simulation results are consistent with the empirical regressions with both trade and FDI.

JEL Classifications: F15, F23, F44.

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\*I am very grateful to my advisors Kei-Mu Yi, German Cubas, and Bent Sorensen for their valuable inputs and guidance throughout this research. Thanks also go to Radek Paluszynski, François de Soyres, Ana Maria Santacreu, Liliana Varela, Arpita Chakravorty, Sergiy Kasyanenko, and Elizabeth Luh for their suggestions and comments, as well as to participants at the Texas Macroeconomic Student Conference and the University of Houston macroeconomic student workshop series. I thank George Alessandria and Horag Choi for sharing their code, and Felix Tintelnot for sharing results from their working paper [Gumpert et al. \[2017\]](#). All remaining errors are mine.

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# 1 Introduction

Is international trade an important contributor to business cycle comovement across countries? While the existing empirical literature has established a strong and positive association between the two, the standard international real business cycle (IRBC) model has failed to generate the observed strength of the correlation. This theoretical shortcoming is called the “trade-comovement puzzle” (Kose and Yi [2006]).<sup>1</sup> Subsequent research has introduced additional features into the standard model to solve the puzzle, with varying degrees of success.<sup>2</sup>

It is of course well known that international trade has grown enormously in the last three decades. It is also well known that during this period multinational production (MP) and foreign direct investment (FDI) have grown at even faster rates. World affiliate sales grew eight-fold from 7% of world GDP to 58% between 1990 and 2007 (Ramondo [2014]). World exports, on the other hand, rose only from 19% of world GDP to 31% during the same period. Moreover, FDI and trade are highly correlated across country-pairs.

In this paper, I investigate empirically and theoretically the joint relationship between trade, FDI, and business cycle comovement. In the empirical part, I conduct the standard fixed effects trade-comovement regression by also including FDI as a right-hand side variable. In the regression, I find that FDI is economically and statistically significant, while the coefficient on trade is considerably smaller. I then build a model of trade and FDI with heterogeneous firms in a dynamic business cycle setting. Drawing from Melitz [2003] and Helpman et al. [2004], I model firms as producing distinct varieties with heterogeneous productivities. Firms choose whether to produce domestically, and also whether to serve foreign markets. If they choose to serve foreign markets, they also choose whether to serve such markets by exporting or by multinational production. Multinational production is subject to a technology spillover from the parent to the

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<sup>1</sup>Kose and Yi [2006] show that a typical country pair in their sample experienced an increase in comovement of 0.06 when trade doubled at the median. However, Backus et al. [1994], the standard IRBC model, generates a relationship that is ten to twenty times weaker.

<sup>2</sup>see Johnson [2014], Liao and Santacreu [2015], Drozd and Nosal [2008], de Soyres [2016].

affiliate. Finally, firms are affected by aggregate productivity shocks, and there is capital accumulation over time, as well as endogenous labor. I then calibrate and simulate the model to match key features of the intensive and extensive margins of trade and FDI. The simulations replicate the pattern found in the empirical regressions.

My empirical analysis includes twenty OECD economies covering the period 1993-2012. I estimate the average effect of trade and multinationals on comovement within country-pairs by including bilateral fixed effects.<sup>3</sup> The fixed effects control for time invariant unobservables, such as contiguity and distance, that might jointly affect trade, FDI, and comovement. When I regress comovement against trade only, doubling the trade share is associated with an increase in comovement of 0.23.<sup>4</sup> When I regress comovement against FDI only, doubling FDI is associated with an increase in comovement of 0.05. When I regress comovement on both variables, however, the trade coefficient falls by nearly a half and is not significant, while the FDI coefficient is relatively unchanged and remains significant. I aim to match these empirical patterns with my calibrated model.

I develop a two-country dynamic model with heterogeneous firms, trade, and multinational production. Within each country, in each period, the number of firms is determined endogenously such that the conditional expected profit equals the sunk cost of entry. Firms' idiosyncratic productivity is drawn from an exogenous distribution after paying the sunk cost. Upon entry, all the firms serve the domestic market; in addition, they can choose to serve the foreign market. For the latter firms, there is an additional choice - either produce at home and export, or set up an affiliate abroad and serve locally. These modes involve different cost structures. Exporting is characterized by a lower fixed cost compared to setting up an affiliate, but it involves an "iceberg" shipping cost that is proportional to the quantity shipped. Because more productive firms are more profitable, in equilibrium, only the firms above a threshold productivity will export. However, exporting is not the profit-maximizing option for the most productive firms. These firms

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<sup>3</sup>I use contemporaneous correlation of cyclical component of quarterly real gross domestic product as the baseline measure of comovement.

<sup>4</sup>The mean of the comovement measure is 0.4; so the 0.23 increase in correlation is over 50% of the mean.

sell large quantities abroad, so they face large export shipping costs. Therefore, they are better off serving the foreign market by setting up an affiliate abroad, despite the higher fixed cost involved. As a result, in equilibrium, the firms with the least productivities only serve the domestic market, the firms with intermediate productivities export, and the most productive firms become multinationals. The capital employed by affiliates of multinationals, evaluated at current investment prices, represents the stock of FDI in each country.

Transitory aggregate productivity shocks generate business cycles in each country. These shocks are modeled as shifters on firms' productivity, such that, when the home country gets a positive aggregate productivity shock, it affects every firm producing in home. In addition, because multinationals transfer technology from their parents to affiliates, the home aggregate shock affects the affiliates located abroad. I model technology transfer such that a fraction of affiliate productivity is transferred from its parent, albeit with an MP "iceberg" cost. A higher MP iceberg cost implies lower efficiency in transferring technology across borders. To close the model, I assume that the households own the capital stock in each country, and make consumption/savings and labor/leisure decisions under the constraint of financial autarky.

The numbers of total firms, exporters, and multinationals are determined period-by-period by the aggregate macroeconomic variables in both the countries. In particular, the aggregate productivity shocks generate business cycles in each country, which determine firms' responses. Exporters' and multinationals' choices in turn result in cross-border shock-spillovers, affecting output, trade, and FDI. In a world with greater trade or FDI shares, these propagation channels are stronger, which results in greater comovement.

The model includes three mechanisms of comovement: first, there are demand-supply spillovers similar to the standard model; second, shocks propagate via multinationals because they transfer technology; and third, the entry/exit of exporters and affiliates at business cycle frequencies generates comovement. The demand-supply spillover channel works in the following way: a shock to one country's aggregate productivity raises output of the final good in that country, which raises demand for foreign varieties. Some of this

higher demand is met by foreign exporters, so the foreign output increases. Like in the standard model, this remains a quantitatively weak propagation channel.

The second channel arises due to the assumption that multinationals transfer their productivity from parents to affiliates. The strength of this mechanism is governed by two factors: first, the extent to which multinationals transfer technology, and second, the share of output produced by multinational firms. An increase in either of these leads to greater comovement of business cycles.

Third, variation in number of exporters and affiliates over business cycles also affect comovement. When one country gets a positive productivity shock, it exports more varieties, which increases the final good output abroad due to the love-for-variety effect embedded into the final good production. In addition, I find that a positive shock to home aggregate productivity leads to an increase in the number of home-owned affiliates producing abroad. This directly expands the foreign GDP. The number of affiliates increases due to two reasons. First, the MP cutoff falls in home- the ex-ante most productive home exporters stop exporting, and enter multinational activity by setting up affiliates abroad. They do so to save on the cost of production- factor prices increase (in relative terms) in home as a result of their higher marginal productivities, in response to the positive shock. Second, the aggregate shock increases conditional expected profits in home, resulting in more firm entry, which further amplifies the number of home-owned affiliates producing abroad.

To evaluate the model, I simulate 200 artificial country-pairs by employing a two-step calibration procedure.<sup>5</sup> First, I calibrate all the parameters to USA-Rest of the World (RoW) pair, where RoW is defined as the composite of countries other than the USA in my empirical regressions. In the second step, I vary the iceberg costs of trade and MP jointly to generate variation in trade and FDI shares. Each combination of the iceberg costs corresponds to one “artificial country-pair.” I simulate the model for each country-pair, and record the resulting trade and FDI shares, and the comovement measure. With

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<sup>5</sup>The number of country-pairs simulated roughly matches the number of unique country-pairs (182) in my sample. My simulation results do not change if I simulate more or fewer pairs. Also, note that all calibrations match the desired moments in the deterministic steady-state of the model.

the simulated data, I run a regression of comovement against trade and FDI separately, and against both variables, as I do in the empirical exercise. I find that the model can generate the positive and significant relationship between trade and comovement, and between FDI and comovement, as observed in the actual data. When both variables are included in the regression, the trade coefficient falls to one-third of its value in the trade-only regression. The FDI coefficient is relatively unchanged compared to the FDI-only regression. These patterns match qualitatively the results from my regressions with the actual data.

Referring back to the three channels of comovement, the shock propagation channels that operate via trade are quantitatively weaker compared to the channels that operate via multinationals. This is the reason why the FDI-comovement relationship remains stable in the model regressions, compared to the trade-comovement relationship. In response to a home aggregate shock, an increase in home affiliates abroad raises the foreign GDP directly; but greater number of home exporters increases foreign GDP indirectly- the love-for-variety affect first increases the output of foreign final good, which then increases the demand for foreign intermediates, and the foreign GDP expands. In addition, technology transfer creates a direct link between the level of shock spillover and the prominence of multinationals in production.

This paper is related to three strands of literature. The first is the empirical research seeking to understand the trade-comovement relationship after controlling for bilateral MP linkages. [Jansen and Stokman \[2014\]](#) regress comovement against trade and bilateral FDI stock for a cross-section of 88 OECD country-pairs. They conclude that FDI was the primary contributor to comovement after 1995. [Kleinert et al. \[2015\]](#) use the variation in bilateral multinational production (MP) and trade between 21 regions in France and their foreign partners. They build more robust measures of MP, compared to FDI stock, using firm level employment and ownership data aggregated to bilateral region-foreign partner level. While trade and MP individually contributed to comovement, they too find that MP was more robustly associated with comovement compared to trade.

The second is towards the quantitative theory research on trade and comovement.

Kose and Yi [2006] showed that the standard international business cycle model (Backus et al. [1994]) generated only one-tenth of the observed trade-comovement relationship. The weakness of the standard model is explained by the Kehoe and Ruhl [2008] result. With competitive output markets, Kehoe and Ruhl [2008] show that a shock to terms of trade affects the real GDP of a small open economy only through changes in the factors employed, and that the shock has zero first order effect on productivity.<sup>6</sup> Kose and Yi [2006] show that factor employment responds only weakly to changes in trade shares. Since then several papers have tried to match the observed relationship between trade and comovement (see Burstein et al. [2008], Liao and Santacreu [2015], de Soyres [2016], Johnson [2014] and Drozd and Nosal [2008]). de Soyres [2016] combines the additional channels proposed in Burstein et al. [2008] and Liao and Santacreu [2015], including international production sharing, firm heterogeneity, and monopolistic competition, to a multi-country version of the standard model. In addition, he allows for the possibility that firms can be both importers and exporters at the same time. His model can generate 70% of the observed trade-comovement relationship. To the best of my knowledge, this is the only paper that can generate high trade-comovement slope in a model without FDI. Drozd and Nosal [2008] take a different approach to the puzzle. They build rigidities in their model to generate difference between short and long run price elasticity of trade. In comparison, I generate additional comovement in a dynamic business cycle model with heterogeneous firms, trade and MP. My model is also an extension of Alessandria and Choi [2007] and Ghironi and Melitz [2005] frameworks, to which I include multinational production.

Third is the empirical literature on multinationals and comovement. See, for example, Budd et al. [2005], Buch and Lipponer [2005], Desai et al. [2009], Desai and Foley [2006], di Giovanni et al. [2015], Cravino and Levchenko [2016], Kleinert et al. [2015], and Boehm et al. [2015]. In particular, Cravino and Levchenko [2016] use firm-level ownership data

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<sup>6</sup>Main reason for the Kehoe and Ruhl [2008] result is the way statistical agencies measure real GDP. In particular, with chain-weighted real GDP, the "base period" is always the preceding year. Because the final output production is perfectly competitive, the change in its output in response to terms of trade shock always equals the change in import bill. On the net, therefore, real GDP does not change when factors are fixed.

for multiple countries and find evidence for technology transfer among multinationals' parent and affiliate entities. [di Giovanni et al. \[2015\]](#) also use a micro firm-level dataset to disentangle the relative contributions of trade and multinational linkages at the firm level. I draw from [Cravino and Levchenko \[2016\]](#) in how I model technology transfer between multinational parents and affiliates.

The rest of the paper is organized as follows- section 2 provides empirical evidence on relationship between trade and comovement when FDI is included in the regression; section 3 proposes a dynamic business cycle model with heterogenous firms, exporters and multinationals; section 4 details the calibration procedure; section 5 provides the results and intuition, and section 6 concludes.

## 2 Empirical Evidence

In this section, I reassess empirical evidence on the correlation between trade and business cycle comovement. As mentioned in the literature review, a number of recent papers have shown that multinationals are quantitatively important channels for cross-country shock spillovers, using firm-level datasets. However, the literature on the aggregate relationship between trade and comovement has not incorporated it into the analysis. It is therefore possible that the trade-comovement slope was overestimated. I am interested both in how the trade-comovement slope changes when FDI is controlled for, as well in the FDI-comovement slope itself. I use FDI stock as a proxy for multinational activity because aggregate bilateral data on the share multinationals' value added in GDP, or the share of multinational employment in total employment are not available for many countries, and when available, the period is severely deficient to undertake any meaningful analysis.<sup>7,8</sup>

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<sup>7</sup>Data on multinationals' employment, sales at bilateral levels are available between 2007-12. However, there are many missing values. An alternative approach is undertaken by [Kleinert et al. \[2015\]](#), who exploit variations in bilateral trade and MP between 21 French regions and their international partners.

<sup>8</sup>Stock of FDI is defined as the aggregate of all investments owned by foreign entities/persons. An investment in a firm is recorded as FDI if a foreign investor intends to have 'lasting interest' in that firm. According to Benchmark Definition 3 (BMD3), which is an OECD standard for recording financial flows, an arbitrary ownership threshold of 10% is used to record FDI data where any foreign investment which results in an ownership share above this threshold is recorded as FDI. Stock of FDI is therefore a measure of capital stock owned by foreign entities with lasting interest. Under reasonable assumptions on firm production function, FDI stock therefore serves as a reasonable proxy for multinational value added.



My main estimating equation is,

$$Corr(Y_{i\tau}, Y_{j\tau}) = \alpha + \beta \log(Trade_{ij\tau}) + \gamma \log(FDI_{ij\tau}) + \delta_{ij} + \epsilon_{ij\tau} \quad (1)$$

where  $Corr(Y_{i\tau}, Y_{j\tau})$  are measures of business cycle synchronization between countries  $i$  and  $j$  during time period  $\tau$ ;  $Trade_{ij\tau} = \max\{\frac{Trade_{ij\tau}}{GDP_{i\tau}}, \frac{Trade_{ij\tau}}{GDP_{j\tau}}\}$  is a measure of trade intensity between countries  $i$  and  $j$ ;  $FDI_{ij\tau} = \max\{\frac{FDIstock_{ij\tau}}{GDP_{i\tau}}, \frac{FDIstock_{ij\tau}}{GDP_{j\tau}}\}$  is a measure of MP between countries  $i$  and  $j$ , and  $\delta_{ij}$  are country-pair fixed effects.

In the baseline estimation, I use the correlation of detrended (cyclical component of real-GDP series detrended using HP filter) quarterly, real GDP as a measure of comovement and a time period,  $\tau$ , equals ten years. For trade and FDI shares, I divide total trade by their respective GDPs and take the total trade as a share of smaller country's GDP. For each time period, I take the average of the trade and FDI shares over the 10 years (see [de Soyres \[2016\]](#)). This approach takes into account the fact that larger countries drive the comovement, while smaller countries respond to larger countries' business cycles. For a given level of total trade, for example, shocks affect smaller countries more because trade is a larger part of their GDP. For robustness, I use other measures of trade and FDI shares.

Data on quarterly real GDP is available from the OECD database. I use this series to compute three measures of comovement. The first two measures are correlations of cyclical components of HP filtered (with smoothing parameter 1600) and Baxter-King filtered series, respectively. The BK band pass filter is set to identify fluctuations between 32 to 200 quarters, reflecting medium term business cycles. The third comovement measure is the correlation of real output growth rate across countries. In the main regression table, I provide results with correlations HP filtered GDP and the correlation of GDP growth rates. In the appendix, I show that the main results are robust to using the BK filtered GDP.

To calculate measures of trade and FDI stock, I use bilateral trade and FDI data from UN COMTRADE database and OECD respectively. The FDI stock data is available from 1985-2013, but I do not use any years prior to 1993 because of a large number of missing

values. I also drop the year 2013 due to the same reason. In the baseline analysis, available period between 1993-2012 is split into two non-overlapping periods of 10 years each, which serve as two time periods (denoted by  $\tau$ ). I use all country-pairs that have at least one non-missing value for FDI in each of the 10-year periods in my regression and provide results with fewer data-points as robustness checks in the appendix. With 20 country-pairs, a maximum number of 190 combinations per time period,  $\tau$ , are possible because comovement, trade, and FDI measures are all symmetric. Among these combinations, eight country-pairs have FDI instock or outstock missing for all years in at least one time period. The remaining 182 pairs, for two time periods (1993-2002 and 2003-2012) constitute the 364 observations in [Table 1](#).

Before I proceed to the results, I present a graphical analysis of the relationships between trade, FDI and comovement in my sample. [Figures 1a and 1b](#) show these relationships during 1993-2012. Each data point in [Figures 1a and 1b](#) represents a country-pair-time-period combination, so a given country-pair appears twice. Figure 1a shows that the FDI-comovement relationship is positive. At the same time, country-pairs with significant FDI linkages also trade more, as seen in [Figure 1b](#). Together, these suggest a positive bias in the prior estimates of the trade-comovement slope. The results from [Table 1](#) confirm this hypothesis.

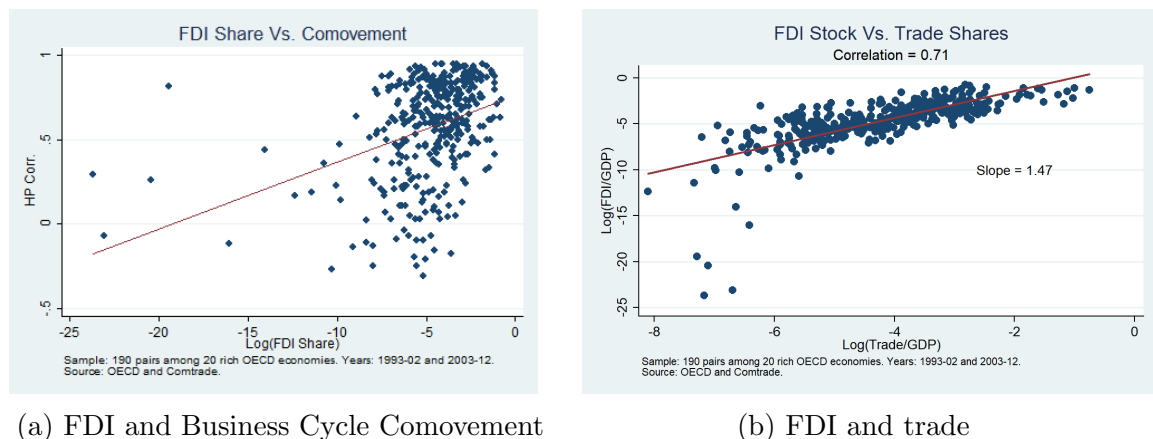


Figure 1: Relationships between FDI, Trade and Business Cycle Synchronization in my sample. Trade and FDI intensities are measured as in equation 1.

In columns 1-3 of [Table 1](#), I display results from regressing baseline measures for comovement against trade and FDI shares. There are three important findings. First,

in line with existing literature, I find that within country-pairs, trade and comovement are positively related. The point estimate in column 1 (0.32) implies that a doubling of trade shares between two countries is associated with an increase in the correlation of their business cycles by 0.22 on average. Second, I find that FDI is also positively associated with comovement. Doubling of FDI share between two countries in my sample is associated with an increase in comovement of 0.05. Third, in column 3, I find that the trade-comovement relationship falls by a half when FDI is controlled for. However, the FDI coefficient itself falls slightly and remains significant.

Table 1: Empirical Regressions - Results

	(1)	(2)	(3)	(4)	(5)	(6)
Log(Trade)	0.32** (0.10)		0.18 (0.10)	0.25* (0.10)		0.12 (0.10)
Log(FDI)		0.07*** (0.02)	0.06*** (0.02)		0.07*** (0.01)	0.06*** (0.02)
Country-pair FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	364	364	364	364	364	364
$R^2$	0.471	0.507	0.515	0.483	0.518	0.522

Standard errors in parentheses

In columns 1-3, I use the baseline comovement measure- correlation of cyclical component of HP filtered real GDP. Columns 4-6 use the correlation of GDP growth rates. Sample: 182 country-pairs between 20 OECD economies per time period, and two time periods covering 1993-2012.

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

The results I find are qualitatively in agreement with the conclusions from [Jansen and Stokman \[2011\]](#), [Kleinert et al. \[2015\]](#). I find that their results are robust within country-pairs and hold for a larger set of countries. The results indicate that most of the increase in comovement over the past three decades can be attributed to multinationals, rather than trade, as was hypothesized before.

**Robustness Checks:** In the first set of robustness exercises, I use alternate measures of comovement. Correlation of cyclical components of Baxter and King band-pass filtered GDP, and the correlation of GDP growth rates are two such measures that are widely used in the IRBC literature. Columns 4-6 in [Table 1](#) display results with the latter

measure for comovement. For the band pass filter, I set the fluctuations between 32 to 200 quarters to identify medium-term business cycles. [Table 9](#) reports results from using the BK filtered GDP measure. The general pattern described above hold with these alternative comovement measures as well.

The results from baseline estimation are also robust to alternate definitions of time windows and the exclusion of country-pairs with more missing values. I implement an alternative specification using 21 years during 1992-2012 split into three non-overlapping seven-year intervals. [Table 10](#), column 4 suggests that choice of time intervals is not driving the results: the changes in coefficient magnitudes between trade-only and FDI-only regressions and the full specification are comparable to the baseline results.

Finally, the inclusion of financial crisis years (2008-2012) in the baseline specification may be a cause for concern. [Table 11](#) in the appendix reports estimates from excluding these years from the analyses. In these estimates too, the general pattern described above remains.

### 3 Model

I develop a dynamic business cycle model with heterogeneous firms, trade and multinational firms. The heterogeneous firm framework draws from [Melitz \[2003\]](#), as well as [Helpman et al. \[2004\]](#). There is endogenous entry and exit such that unconditional expected profit is driven to zero. Monopolistically competitive firms vary by productivity, and choose whether to serve foreign markets, and if so, whether to do it by exporting or by multinational production. The dynamics draw from the international real business cycle literature and includes aggregate total factor productivity shocks, capital accumulation, and endogenous labor supply.

**Households:** Atomistic households within each country maximize their lifetime expected utility by optimizing intertemporal consumption behavior based on the information available at any given time. In addition, households optimize labor supply decision depending

on the opportunity cost of leisure. I assume that current account is balanced every period and the households optimize decisions under financial autarky. Households' utility function is given by,

$$U_{it} = \max_{C_{it}, L_{it}, K_{i,t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_{it}) - \psi \frac{L_{it}^{1+\nu}}{1+\nu} \right]$$

Such that  $C_{it} + (K_{i,t+1} - (1 - \delta)K_{it}) = w_{it}L_{it} + r_{it}K_{it}$ ;  $w_{it}$  and  $r_{it}$  are real wage and real rental rate respectively in country  $i$ , and  $\delta$  is the depreciation rate of capital. The parameter  $\beta$  determines 'patience' of the households,  $\psi$  governs the preference for leisure, and  $1/\nu$  is the Frisch-elasticity of labor supply. The intertemporal elasticity of substitution is assumed to be one.

### Firms:

**Final Good Producer:** Within each country, the final good producer aggregates all of the different product varieties available within that country without adding any value in the process. The final good is assumed to be non-traded and must be consumed or invested by households within that country. The final good production function takes the following constant elasticity of substitution (CES) form,

$$Y_{it} = \left[ \int_{\omega \in \Omega_{it}} q_{i,t}^{\frac{\sigma-1}{\sigma}}(\omega) d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where  $\Omega_{it}$  is the endogenous set of varieties  $\omega$  available in country  $i$  at time  $t$ . Given the aggregate prices and the prices of intermediate producers, demand for a variety from the final good producer is,

$$q_{i,t}(\omega) = \left( \frac{p_{i,t}(\omega)}{P_{i,t}} \right)^{-\sigma} Y_{i,t}$$

and the aggregate price index of the consumption/investment bundle is,

$$P_{it} = \left[ \int_{\omega \in \Omega_{it}} p_{i,t}^{1-\sigma}(\omega) d\omega \right]^{\frac{1}{1-\sigma}}$$

**Intermediate Goods Producers:** Within each country, there exist an infinite number

of potential intermediate producers, but only an endogenous subset of these firms produce during any period. This number is determined by a free entry condition: in equilibrium, conditional expected profits on entry equals a sunk cost paid to enter. Upon paying the sunk cost, firms draw their idiosyncratic productivity from an exogenous distribution. Therefore, intermediate firms are heterogeneous in their productivity. Given that a firm has chosen to produce, it can do so without any fixed cost in the domestic market. In addition, firms can serve the foreign market by exporting from home, or by setting up an affiliate and producing abroad. If they decide to export, they incur a fixed cost  $F^X$ , and an “iceberg” shipping cost,  $\tau > 1$ . The iceberg cost is standard and takes the following form: if a firm wants one unit of its good to arrive abroad, it has to ship  $\tau$  units from home. A higher  $\tau$  is associated with greater trade friction. Setting up an affiliate requires a higher fixed cost,  $F^M > F^X$ , but the iceberg cost of transportation is foregone. This captures the “proximity-concentration” trade-off that is well known in the literature: firms can either choose concentrate their production in one location to take advantage of economies of scale, or they can split production and be closer to the intended market. All the fixed costs are paid in “efficiency input-bundle” of the country where production takes place.

To distinguish between multinational affiliates from other firms, I use subscripts  $i$  and  $k$  to indicate countries of origin and operation, respectively. Cases where  $i$  and  $k$  are not equal refer to affiliates. Intermediate firms’ production function is written as,

$$Y_{ik,t}(\varphi) = Z_{ik,t}(\varphi)K_{ik,t}^\alpha(\varphi)L_{ik,t}^{1-\alpha}(\varphi)$$

where  $Z_{ik,t}(\varphi)$  is the productivity of a firm with an idiosyncratic productivity  $\varphi$ , originating in country  $i$  and operating in country  $k$ ;  $K_{ik,t}(\varphi)$  and  $L_{ik,t}(\varphi)$  are firm specific capital and labor respectively; and  $\alpha$  is the share of capital in firms’ operational expenditure.

I model firms’ productivity,  $Z_{ik,t}(\varphi)$ , as Cobb-Douglas aggregation of their home and host aggregate productivities and its own idiosyncratic productivity. This characterization draws from [Cravino and Levchenko \[2016\]](#).

$$Z_{ik,t}(\varphi) = \frac{Z_{it}^\mu Z_{kt}^{1-\mu} \varphi}{h_{ik}}$$

where  $Z_{it}$  and  $Z_{kt}$  are home and host specific aggregate productivities, and  $0 \leq \mu \leq 1$  is the fraction of affiliates' productivity that is transferred from its parent. This structure provides an additional channel for propagation of shocks across countries: when  $\mu > 0$ , aggregate shock in one country affects all of its affiliates producing abroad.  $h_{ik} \geq 1$  is the "efficiency iceberg cost of MP," such that  $h_{ii}$  is normalized to 1. In percentage terms, the efficiency iceberg cost can be written as  $1 - 1/h$ : when  $h = 1$ , the cost is 0%; when  $h = \infty$ , cost is 100%. Note that for domestic producers and exporters,  $i = k$ , thus the productivity term above collapses to the home country aggregate productivity and firms' idiosyncratic productivity.

Let  $mc_{i,t} = \frac{Ar_{i,t}^\alpha w_{i,t}^{1-\alpha}}{Z_{i,t}}$  denote the real effective marginal cost of an input bundle in country  $i$ .<sup>9</sup> Since firms operate in a monopolistically competitive market, their prices are equal to a constant markup times the marginal cost of production. For domestically sold varieties, price as a fraction of the home aggregate price is therefore  $\rho_{i,t}^D(\varphi) = \frac{\sigma}{\sigma-1} \frac{mc_{i,t}}{\varphi}$ ; exporters' price (as a fraction of the foreign aggregate price) is  $\rho_{ij,t}^X(\varphi) = \frac{\sigma}{\sigma-1} \frac{mc_{i,t} \tau}{\varphi Q_{ij,t}}$  where  $Q_{ij} = P_{j,t}/P_{i,t}$  is  $i$ 's real exchange rate; and price charged by multinational affiliates is  $\rho_{ik,t}^M(\varphi) = \frac{\sigma}{\sigma-1} \frac{mc_{k,t} h}{\varphi} \left( \frac{Z_{k,t}}{Z_{i,t}} \right)^\mu$ . Using demand for intermediates, a firm's (nominal) profit from operations for each activity, conditional on choosing to engage in that activity, can be written as,

$$\begin{aligned} \Pi_{i,t}^D(\varphi) &= \frac{R_{i,t}^D(\varphi)}{\sigma} \\ \Pi_{ij,t}^X(\varphi) &= \frac{R_{ij,t}^X(\varphi)}{\sigma} - P_{i,t} mc_{i,t} F^X \\ \Pi_{ik,t}^M(\varphi) &= \frac{R_{ik,t}^M(\varphi)}{\sigma} - P_{k,t} mc_{k,t} h \left( \frac{Z_{k,t}}{Z_{i,t}} \right)^\mu F^M \end{aligned}$$

where  $R_{i,t}^D(\varphi) = (\rho_{i,t}^D(\varphi))^{1-\sigma} E_{i,t}$  is the revenue from domestic activities;  $R_{ij,t}^X(\varphi) = (\rho_{ij,t}^X(\varphi))^{1-\sigma} E_{j,t}$  is the revenue from exporting; and  $R_{ik,t}^M(\varphi) = (\rho_{ik,t}^M(\varphi))^{1-\sigma} E_{k,t}$  is the revenue from MP, given consumption and investment expenditures in the destination

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<sup>9</sup> $A = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$

market,  $E_{k,t}$ .

Since firms do not pay a fixed cost to produce domestically, every firm that pays the sunk cost ends up producing. The export cutoff, on the other hand, is given by a zero-cutoff-profit condition:

$$\Pi_{ij,t}^X(\varphi_{ij,t}^X) = 0 \quad (2)$$

where  $\varphi_{ij,t}^X$  is the export cutoff from country  $i$  to  $j$  at time  $t$ . Firms setup an offshore affiliate if the profit from affiliate operations is higher when compared to the export profit. For a marginal affiliate, these profits are equal:

$$\Pi_{ik,t}^M(\varphi_{ik,t}^M) = \Pi_{ik,t}^X(\varphi_{ik,t}^X) \quad (3)$$

where  $\varphi_{ik,t}^M$  is the MP cutoff from country  $i$  to  $k$  at time  $t$ .

**Assumption:** MP costs are sufficiently larger than export costs in all periods, i.e.,

$$\frac{F^M h^\sigma}{F^X \tau^{\sigma-1}} > \left[ \frac{mc_{it}}{Q_{ik,t} mc_{kt}} \left( \frac{Z_{it}}{Z_{kt}} \right)^\mu \right]^\sigma$$

leading to higher MP cutoff in all periods ( $\varphi_{ik,t}^M > \varphi_{ik,t}^X$ ).

Proof: [see appendix](#).

### Aggregate Productivity

I assume that the aggregate productivity shocks are uncorrelated across countries. In this way, I focus on the comovement generated via the endogenous channels in the model. In each country, the aggregate productivity follows an AR1 process in logs,

$$\log(Z_{i,t}) = \rho \log(Z_{i,t-1}) + \epsilon_{i,t}$$

where  $\epsilon_{i,t} \sim N(0, \sigma_\epsilon^2)$ .

### Aggregation

I assume that firms' idiosyncratic productivity is distributed Pareto with a pdf given by  $g(\varphi) = \gamma \varphi^{-\gamma-1}$ . The inherent assumption here is that  $\varphi$  can not be lower than 1, a condition which I enforce when simulating the model. I now calculate the bilateral aggregates



for export and MP revenues and profits which are then used to specify the current account balance and the free entry conditions. Similar to Melitz [2003], aggregate variables in this model can be written in terms of the average productivity for each activity. Let  $R_{ij,t}^X = M_{ij,t}^X R^X(\tilde{\varphi}_{ij,t}^X)$  be the aggregate revenue of all firms from country  $i$  earned from exporting to  $j$ . Similarly,  $\Pi_{ik,t}^M = M_{ik,t}^M \Pi^M(\tilde{\varphi}_{ik,t}^M)$  is the aggregate profit of all affiliates from country  $i$  operating in  $k$ .<sup>10</sup>  $\tilde{\varphi}$  denotes the average of the firms' idiosyncratic productivities in any given activity.  $M_{ij,t}^X$  and  $M_{ik,t}^M$  denote respectively the mass of exporters and multinationals from  $i$ . These expressions for aggregate revenue and profit allow me to write down the free entry and current account balance conditions.

### Equilibrium Conditions

**Current account balance:** In a model of trade and MP, imposing the current account balance condition leads to the assumption that affiliates' profits are repatriated (Arkolakis et al. [2013]). In this way, there is no net transfer of resources across countries. Mathematically, the current account balance condition can be written as,

$$R_{ik,t}^X + \Pi_{ik,t}^M - R_{ki,t}^X - \Pi_{ki,t}^M = 0 \quad (4)$$

**Free Entry:**

$$\Pi_{it} = M_{it} \frac{P_{it} w_{it}}{Z_{it}} F^E \quad (5)$$

where  $\Pi_{it} = \Pi_{i,t}^D + \Pi_{ik,t}^X + \Pi_{ik,t}^M$  is the total (nominal) profits of firms originating in country  $i$  earned in all markets across all activities, and markets, and  $M_{it}$  is the mass of entrants in country  $i$ .

**Goods Market Equilibrium:**

$$C_{it} + X_{it} = Y_{it} \quad (6)$$

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<sup>10</sup>See appendix for expressions for average productivities and number of firms by activity. Average productivity for a given activity is conditional on cutoff bounds on that activity, while number of firms is a function of number of potential entrants and cutoffs. Similar identities hold for aggregate revenues from domestic and multinational affiliates, but are not shown here.

where  $X_{it} = K_{i,t+1} - (1 - \delta)K_{it}$  represents households' investment in country  $i$ .

Firms' demand for capital:  $K_{i,t}^d = K_{i,t}^D + K_{ik,t}^X + K_{ki,t}^M$

$$= \frac{\alpha(\sigma - 1)}{r_{i,t}P_{i,t}}(\Pi_{i,t}^D + \Pi_{ik,t}^X + \Pi_{ki,t}^M) + \alpha\sigma \frac{m_{C_{i,t}}}{r_{i,t}} \left( F^X M_{ik,t}^X + hF^M \left( \frac{Z_{i,t}}{Z_{k,t}} \right)^\mu M_{ki,t}^M \right) \quad (7)$$

Firms' demand for labor:  $L_{i,t}^d = L_{i,t}^D + L_{ik,t}^X + L_{ki,t}^M + L_{i,t}^E$

$$= \frac{1 - \alpha}{\alpha} \frac{r_{i,t}}{w_{i,t}} K_{i,t}^d + \frac{M_{i,t} F^E}{Z_{i,t}} \quad (8)$$

**Equilibrium:** Equilibrium in a two-country system consists of quantities consumption  $\{C_{it}\}_{i,t}$ ; investment  $\{X_{it}\}_{i,t}$ ; final output  $\{Y_{it}\}_{i,t}$ ; labor supply  $\{L_{it}\}_{i,t}$ ; stock of capital  $\{K_{it}\}_{i,t}$  mass of entrants  $\{M_{it}\}_{i,t}$ ; mass of producers, exporters and multinationals  $\{M_{it}^D\}_{i,t}$ ,  $\{M_{ik,t}^X\}_{i \neq k,t}$ ,  $\{M_{ikt}^M\}_{i \neq k,t}$  respectively; wages and rental rates  $\{w_{it}\}_{i,t}$  and  $\{r_{it}\}_{i,t}$ ; real exchange rates  $\{Q_{ikt}\}_{i \neq k,t}$  and such that,

1. Aggregate prices equate consumption and investment demands to the final good production in each country.
2. Prevailing wages and rental rates bring labor and capital markets to equilibrium in each country.
3. Mass of entrants adjusts to satisfy free entry condition
4. Real exchange rates adjust to bring current account to a balance in each country.

## Computing GDP

The GDP data used in the empirical analyses constructs volume estimates of the GDP at OECD reference year prices. To match this as closely as possible, I transform each nominal component in the expenditure based GDP equation using its respective steady-state price. I also correct for the effect of variety on the model-implied prices. I define

Table 2: **Summary of Model Equations**

Investment FOC	$\frac{1}{C_{i,t}} = \beta \mathbb{E}_t \left[ \frac{1}{C_{i,t+1}} (r_{i,t+1} + 1 - \delta) \right]$
Final Good equilibrium	$C_{i,t} + X_{i,t} = Y_{i,t}$
Capital accumulation	$K_{i,t} = (1 - \delta)K_{i,t-1} + X_{i,t-1}$
Cutoffs by activity (Zero Profit)	See appendix
Average productivities by activity	See appendix
Revenues by activity	See appendix
No. of firms by activity	See appendix
Profits by activity	See appendix
Labor market equilibrium	$L_{i,t}^d = \left[ \frac{w_{i,t}}{C_{i,t}\psi} \right]^{\frac{1}{\nu}}$
Capital market equilibrium	$K_{i,t}^d = K_{i,t}$
Free Entry condition	$\Pi_{i,t} = M_{i,t} \frac{P_{i,t} w_{i,t}}{Z_{i,t}} F^E$
Current Account balance	$R_{ik,t}^X + \Pi_{ik,t}^M - R_{ki,t}^X - \Pi_{ki,t}^M = 0$

$\tilde{P}_{i,t} = [M_{i,t}^D]^{\frac{1}{\sigma-1}} P_{i,t}$ , where  $P_{i,t}$  is the welfare-based price index. Export prices are defined similarly by deflating it with the number of exporting firms.

$$RGDP_{it} = \tilde{P}_i^{ss} \frac{P_{it} C_{it} + P_{it} I_{it}}{\tilde{P}_{it}} + \sum_{j \neq i} \tilde{P}_{ij,X}^{ss} \frac{R_{ij,t}^X}{\tilde{P}_{ij,t}^X} - \sum_{j \neq i} \tilde{P}_{ji,X}^{ss} \frac{R_{ji,t}^X}{\tilde{P}_{ji,t}^X}$$

### Computing FDI Stock

The OECD data on stock of FDI is reported in current US Dollars and represents the stock of capital owned by foreign entities. The model equivalent of FDI stock is therefore measured as the capital employed by affiliates, including the capital used to pay MP fixed cost, evaluated using the price of the composite consumption/investment good in a given country. Therefore, changes in FDI stock across time within a country reflect both real activity of multinationals as well as changes to aggregate prices. Similarly, across countries, differences in levels of FDI stock can be driven by variation in the aggregate prices over their respective business cycles.

## 4 Calibration and Simulation

In this section, I detail the calibration and simulation strategy. Ideally, an N-country model, calibrated to match the relative country sizes, and the extensive and intensive margins of bilateral trade and bilateral FDI linkages, should be used. However, data on the extensive margins of trade and FDI are unavailable for most of the 190 country-pairs in section two. Therefore, I simulate the two-symmetric-country model repeatedly. This approach draws from [Liao and Santacreu \[2015\]](#).

### 4.1 Calibration

In the baseline, I calibrate the parameters to match trade and FDI intensive and extensive margins for the USA-Rest of the World (RoW) pair in the deterministic steady state of the model. Here, I define RoW as the set of countries other than the United States in my empirical sample.

The model has fifteen parameters. These include the consumer preference parameters  $(\beta, \nu, \psi, \sigma)$ ; the capital depreciation rate  $\delta$ ; the production parameters  $(\mu, \alpha)$ ; the Pareto shape parameter  $\gamma$ ; the sunk entry cost  $F^E$ ; the parameters of the aggregate productivity process,  $\rho$  and  $\sigma_\epsilon^2$ , and the iceberg and fixed costs of trade and MP  $(h, \tau, F^X, F^M)$ .

The USA-RoW calibration is done as follows. I set the first eleven parameters based on their values from related literature, while iceberg and fixed costs are set to match the average of extensive and intensive margins of trade and FDI shares for the USA-RoW pair. Each period in the simulations corresponds to one year. Therefore, I set the households' preference parameter  $\beta$  to 0.96. The Frisch-elasticity is set to 1.5 following [de Soyres \[2016\]](#), so  $\nu$  equals 0.67. The leisure preference parameter  $\psi$  equals 6.7 which implies labor supply is 30% of available time in the steady-state, in line with fraction of hours worked for the United States. The depreciation rate of capital,  $\delta$ , is set to 0.1. In the baseline, I shutdown technology transfer among multinationals by setting  $\mu$  equal to zero. The comovement generated in the model, therefore, reflects international spillovers via the endogenous mechanisms. I vary  $\mu$  between 0 and 0.3 to test the robustness

of quantitative results. The share of capital in production,  $\alpha$ , equals 0.36. Elasticity of substitution varies between 2-10 in international business cycle literature. I take a conservative estimate of 3.5 (implying 40% markup over marginal cost) in my baseline analysis. The Pareto shape parameter  $\gamma$  is equal to 5.5. I follow [Liao and Santacreu \[2015\]](#), [Ghironi and Melitz \[2005\]](#) and normalize the sunk cost to 1. As a result, all other fixed costs are interpreted in terms of the sunk cost. I set the persistence of aggregate productivity to 0.9 and the standard error of its innovation process,  $\sigma_\epsilon\epsilon$ , is set to 0.02, similar to [Liao and Santacreu \[2015\]](#).

Table 3: Calibration (USA-RoW Pair)

Parameter	Description	Value
$\beta$	Time preference	0.96
$\nu$	Inverse Frisch-elasticity	0.67
$\psi$	Leisure preference	6.7
$\delta$	Depreciation	0.1
$\sigma$	Elasticity of substitution	3.5
$\alpha$	Capital share	0.36
$\mu$	MP technology transfer	0
$\gamma$	Pareto shape parameter	5.5
$h$	MP efficiency cost	1.4
$\rho$	Persistence of aggregate productivity shock	0.9
$\sigma_\epsilon$	SE, innovation to aggregate productivity	0.02
$\tau$	Iceberg cost	1.5
$F^E$	Sunk entry cost	1
$F^X$	Export fixed cost	0.18
$F^M$	Multinational fixed cost	0.2

That leaves the iceberg and fixed costs of trade and MP to be calibrated for the USA-RoW pair. I set these parameters jointly by targeting the model steady state to match the extensive and intensive margins of trade and FDI for that pair. Data on the fraction of exporting establishments in manufacturing for the US is available from [McCallum and Lincoln \[2016\]](#) and its mean is 34.4% between 1987-2006. Data on the number of affiliates of US firms by foreign country is available from OECD.<sup>11</sup> The fraction of affiliates varied

<sup>11</sup>The data on number of affiliates of US-owned multinationals is available from the OECDs outward activity of multinationals database. The affiliate number is closer to the number of plants or establishments, rather than the number of firms. The data on the number of total establishments by year in the United States is available from US Census Bureaus Statistics of US Businesses database. For the latest version of this data, see: <https://www.census.gov/epcd/susb/latest/us/US-.HTM>.

from 2.3% to 3.2% between 1998-2013, with a mean of 2.7%. Finally, trade and FDI shares between USA and RoW for my sample are 27.6% and 19.1% respectively. It should be noted that I do not target the level of comovement between USA and RoW in my sample. Under the baseline calibration, the comovement is 0.22, compared to 0.82 in the data. My goal is to evaluate if the model can generate the observed changes in comovement, in response to changes in trade and FDI shares. [Table 3](#) reports the parameter values in the baseline calibration.

## 4.2 Simulation

To simulate the model, I generate 200 artificial country-pairs that differ in bilateral trade and FDI shares. To generate this variation, I jointly vary the iceberg costs of trade and MP ( $h$  and  $\tau$ ), while other parameters are held fixed. In addition, the simulated shares mimic the positive correlation between trade and FDI observed in the actual data.<sup>12</sup> To get iceberg cost combinations that preserve trade-FDI slope, I first generate the trade-FDI share combinations that mimic the actual data; then, I recalibrate the model repeatedly to match different trade-FDI combinations in the deterministic steady state, allowing only  $h$  and  $\tau$  to vary. I describe the approach below in greater detail.

I start by generating 50 equally spaced FDI shares between 10% and 30% of GDP, and repeat each of these shares 4-times. This 200-by-1 FDI shares vector provides the basis for generating trade shares around it. I then generate 200 i.i.d. normal errors using the residual standard deviation (equal to  $5.7 \cdot 10^{-3}$ ) from a fixed effects regression of trade against FDI in the actual data. The slope estimate from this regression is 0.06, implying that a one percentage point (pp) increase in FDI share is associated with 0.06 pp increase in trade share. Then, I generate trade shares using the vector of FDI shares, the slope

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<sup>12</sup>Varying only the trade iceberg cost, for example, generates variations in both trade and FDI shares. When shipping is less expensive (i.e., when  $\tau$  lower), trade increases and MP decreases. Therefore, it is essential to vary both  $h$  and  $\tau$  simultaneously to generate the positive slope between trade and FDI. [Liao and Santacreu \[2015\]](#), in their model without MP, vary the fixed and iceberg costs of trade to generate artificial country-pairs. They take a “grid” approach, where they generate combinations of fixed and iceberg costs uniformly distributed around their respective baseline values. In my approach, on the other hand, the values taken by  $h$  and  $\tau$  are governed by the positive relationship between trade and FDI that I generate. Taking uniformly distributed  $h$  and  $\tau$  combinations around their respective baseline values will not generate this data feature.

estimate, and the normally distributed error terms:

$$\text{Trade Sh} = 0.129 + 0.06 \text{ FDI Sh} + \epsilon \quad \text{Std.Err}(\epsilon) = 0.0057$$

I now have a 200-by-2 matrix with trade and FDI shares in the columns. For each country-pair, I target one combination of trade and FDI shares by varying  $h$  and  $\tau$  appropriately. Across the artificial country-pairs, the MP iceberg cost  $h$  varies between 1.44 to 1.62. This implies that affiliates lose 30% to 38% of their productivity when locating abroad. The trade iceberg cost  $\tau$  varies between 1.58 to 1.76. For each of the 200 artificial country-pairs, I run the model 100 times with 20 periods in each run. The FDI and trade shares, and the comovement measures are taken to be the average across the 100 runs. [Figure 6](#) in the appendix provides a comparison between actual and simulated data.

## 5 Results

Using the simulated data for trade and FDI shares and comovement, I run regressions akin to my empirical specifications in columns 1-3, [Table 1](#). [Table 4](#) provides the estimates from these regressions. The first three columns correspond to the baseline case in which I shut down technology transfer among multinationals. In columns 4-6, 30% of affiliates aggregate productivity comes from their parent country (i.e.,  $\mu = 0.3$ ), resulting in direct shock-spillover across borders.

There are three main takeaways from [Table 4](#). First, in columns 1 and 2, the model generates positive and statistically significant relationship between trade and comovement, and FDI and comovement. Second, under the full specification in column 3, the trade coefficient falls to nearly a third of its value in the trade-only regression, while the FDI coefficient is essentially unchanged compared to its value in the FDI-only regression. Both of these results qualitatively match the observed patterns in section two. Third, my simulations generate the relative magnitudes of trade-comovement and FDI-comovement coefficients. For example, in the data, the trade-coefficient in the trade-only regression (column 1, [Table 1](#)) is nearly four times the FDI coefficient in the FDI-only regression

Table 4: Quantitative Exercises - Results

	$\sigma = 3.5$					
	$\mu = 0$			$\mu = 0.3$		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$\log(\text{trade})$	0.33*** (0.05)		0.07*** (0.05)	1.26*** (0.11)		0.38*** (0.05)
$\log(FDI)$		0.08*** (0.01)	0.07*** (0.01)		0.28*** (0.01)	0.25*** (0.01)

Results from estimating trade-comovement regression with model implied numbers for 200 artificial country-pairs. For each country-pair, I run the model 100 times with 20 periods in each simulation. Columns 1-3 present results from a model without technology transfer among multinationals. In columns 4-6, 30% of affiliate productivity comes from its parent.

(column 2, [Table 1](#)). The same pattern is observed in my simulations as well (columns 1 and 2, [Table 4](#)). These patterns are robust to alternate parameter values for the elasticity of substitution between varieties,  $\sigma$ , and the MP technology transfer parameter,  $\mu$ . These robustness analyses are provided in the appendix.<sup>13</sup>

Including FDI generates two additional comovement mechanisms which are not a part of trade-only models: (i) variation in extensive margin of affiliates over business cycle, which directly affects GDP, and (ii) technology transfer among multinationals. Both of these mechanisms are quantitatively important relative to the channels that operate through trade. Trade affects comovement via the demand-supply spillover due to the final good, and via extensive margin adjustment among exports over the business cycle. Both of these mechanisms are quantitatively weaker. I explain the differences between trade-only models and my model in greater detail in section 5.2.1.

When multinationals transfer technology across countries, the coefficient magnitudes are higher. This is evident in columns 4-6 of [Table 4](#). This happens for two reasons: when  $\mu > 0$ , (i) aggregate shocks are transmitted across borders via multinationals, generating greater comovement, and (ii) the MP extensive margin is more volatile over business cycle frequencies. I explain the latter in section 5.2.2.

<sup>13</sup>The results are robust to including bilateral fixed effects as well. To check this, I create random pairs between the 200 simulations, and run a fixed effects regression similar to the one section 2. On average, the coefficient magnitudes do are very similar to [Table 4](#).



## 5.1 Understanding Coefficient Magnitudes

The results presented above show that the model generates qualitatively the observed patterns in how coefficients vary across empirical regressions. But the coefficient magnitudes are relatively high, and are sensitive to the multinationals' technology transfer parameter,  $\mu$ . For example, in column 1 of [Table 4](#), the trade-comovement slope matches the observed value in [Table 1](#). Unfortunately, this does not mean that the puzzle is solved yet. The coefficient magnitudes are relatively large and are sensitive to  $\mu$ , because the volatility of extensive margin of affiliates is about three times higher in my model compared to the data. I now detail this weakness in the model, and explain how it can be resolved by imposing additional structure.

Table 5: Volatility in Affiliate Shares, Model Vs Data

	Model		Data
	Mean	Max	
Residual diff, peak to trough	9.5%	21.3%	6.3%

Difference in residuals from log-linear detrended affiliates shares. In the data, differences are computed between peak and trough in the US. In the model, I average the differences in log affiliate shares between peak and trough over 100 simulations with 20 periods in each simulation.

Greater volatility in multinationals' extensive margins is the main reason why coefficient magnitudes are large. In my model, a large number of ex-ante exporters from the country that gets a positive shock cease to export, and become multinationals by setting up affiliates instead. The shock raises wages and rental rates at home, and these firms benefit from lower relative cost of production abroad. This shift in production location raises foreign GDP significantly, and as a result comovement is relatively large given the trade and FDI shares.

I estimate that the volatility of affiliates lies between 1.5 times to 3 times as large compared to the data. To show this, I compare the volatility under the baseline USA-RoW calibration and in the data for affiliates of US multinationals. I define the volatility

measure as the difference in log affiliate share (affiliate share is the number of affiliates of US owned firms, as a fraction of all US manufacturing establishments), after detrending it, between peak and trough of business cycles in the US. Between 2007 and 2009, which is the only BEA defined peak and trough for which the affiliate share data is available, the difference in affiliate share was 6.3%. I treat this as an upper bound because the Great Recession resulted in a large reduction of economic activity. With the model, then, I simulate the baseline USA-RoW pair 100 times, with 20 periods in each simulation. The average difference in log affiliate share was 9.3% across these simulations, and the maximum was over 21%. Using the latter as a comparable scenario to the 2007-2009 episode, I observe that the volatility in my simulations is about three times as large.

The volatility of affiliate share over business cycles can be calibrated to the data by making multinationals' entry/exit decisions dynamic. [Gumpert et al. \[2017\]](#) show that life-cycle dynamics of multinationals can be better modeled by including sunk costs to multinational production. The sunk cost makes decision to enter or exit multinational activity state dependent: firms that are already multinationals are less likely to exit, and non-multinationals are less likely to enter. Including the sunk cost in my framework will reduce the volatility of affiliate share over business cycle, and therefore the level of comovement will decrease. In this augmented model, the volatility of affiliates and the multinationals' technology transfer parameter,  $\mu$ , together will determine the coefficient magnitudes.

## 5.2 Model Mechanisms - Impulse Responses

I now present a detailed explanation of the model results in [Table 4](#) using impulse responses of relevant variables. These impulse responses are generated for the baseline USA-RoW calibration. In each simulation, one country, which I call Home, gets a one-time, one standard deviation (2% increase) shock to its aggregate productivity ( $Z_H$ ). In subsection 5.2.1 I look at the differences between impulse responses under the baseline calibration and another case with prohibitively high costs of MP; and finally in subsection 5.2.2, I present results under different levels of technology transfer among multinationals.

### 5.2.1 The Role of Multinational Firms

How does a model with MP and trade compare against a model with only trade? To see this, I compare impulse responses under baseline calibration and a model that has prohibitively high MP costs. I recalibrate the latter model to match extensive and intensive margins of trade for USA-RoW pair, while MP margins are set to zero. [Figure 2](#) plots the impulse responses of selected endogenous variables under the two scenarios. The time path of  $Z_H$  is shown at the bottom. The solid blue lines in the figure correspond to the baseline calibration, while the red dashed lines correspond to the high MP costs case.

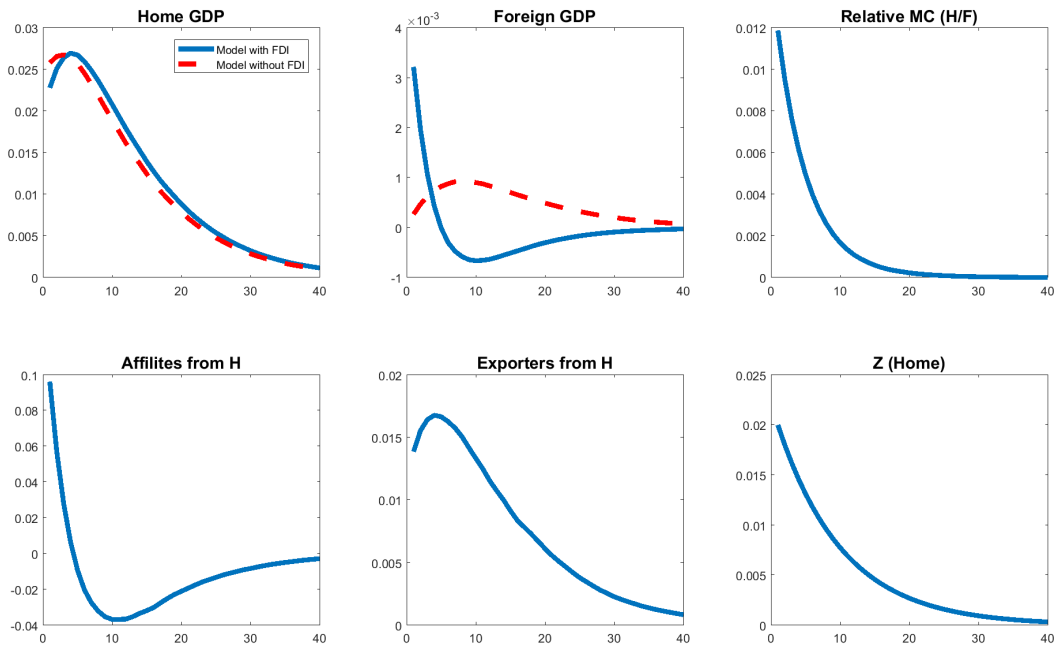


Figure 2: Comparison of impulse responses under models with and without MP.

The main takeaway from [Figure 2](#) is that given a shock to Home aggregate productivity, Foreign GDP increases more (in percentage terms) in a model with multinationals. Immediately upon impact, the trade-only model generates only 7% of the increase in Foreign output compared to a model with MP. The intuition is as follows. When H gets a positive aggregate shock, the relative wages and rental rates increase in H. This results in an increase in the number of home affiliates producing abroad. This happens due to two reasons. First, MP cutoff falls in home due to higher relative rental rates and wages at home. The foreign GDP's weak expansion does not generate sufficient increase in demand

to justify producing at higher costs at home. Note that fixed costs also are lowered by producing abroad, since MP fixed cost is paid in the country of production. Second, the aggregate shock increases flow of profits for home firms, so more firms enter. Together, the reduction in MP cutoff and higher number of firms results in greater affiliates abroad. This channel is absent in a trade-only model.

### 5.2.2 Varying $\mu$

Multinationals' home productivity share,  $\mu$ , is another important model parameter. Higher the value of  $\mu$ , greater is the level of shock spillover, and hence comovement, for a given level of MP. At the same time, I show that higher  $\mu$  also results in greater volatility of affiliates' extensive margin. Higher volatility causes further increase in comovement, given everything else. To show this, I simulate the baseline model under different values of  $\mu$  ( $= 0, 0.3$  and  $0.45$ ) due to the same Home aggregate shock in section 5.2.1.

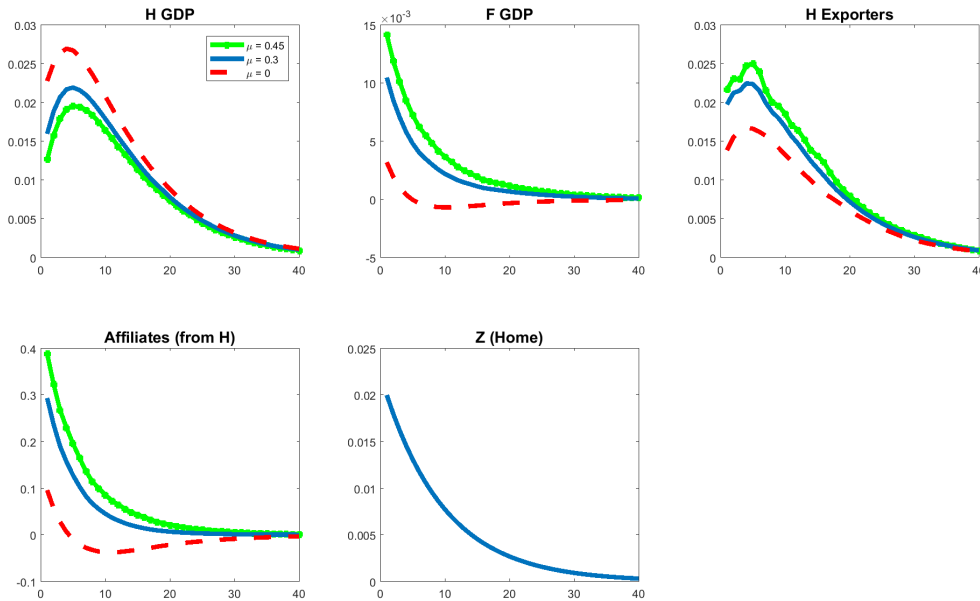


Figure 3: Correlation of real GDP under different values of  $\mu$ .

When H gets a positive aggregate shock, multinational affiliates from H in F carry a share  $\mu$  with them. When  $\mu$  is higher, shocks are transmitted to a larger extent, resulting in higher correlation of GDP (see Figure 3). Also, MP extensive margin is more volatile

under greater  $\mu$ . When  $\mu > 0$ , shock to home productivity provides additional incentives for home exporters to shift production abroad- they get to save on production costs as well as have a greater productivity advantage. When aggregate productivities are symmetric (i.e.,  $Z$ s take the same value in steady state in all countries), changes to  $\mu$  do not change the steady state. All the resulting differences in impulse responses are purely due to variations at business-cycle frequencies. Given a value of  $\mu$ , higher MP in steady state also results in larger comovement.

## 6 Conclusions

Existing literature has established a strong and robust correlation between international trade and business cycle comovement within country-pairs. Several authors have taken this empirical specification as given and attempted to refine the international real business cycle theory to generate the empirically observed trade-comovement slope. Of the different mechanisms employed within these models, the only existing solution includes heterogeneous firms, international input-output linkages, and endogenous markups to generate about 70% of the slope.

In this paper, I show that for rich OECD economies, the empirical slope falls considerably when FDI stock is included in the regression, while FDI coefficient is relatively stable, and is statistically significant. This result is in line with comparable empirical literature and shows that multinational production linkages are quantitatively important for comovement, relative to trade. Empirical trade-comovement regressions that exclude FDI therefore suffer from significant omitted variable bias.

In the main part of this paper I develop a two-country IRBC model with heterogeneous firms, and both trade and FDI linkages. The model includes two mechanisms that are part of the trade-only frameworks- the demand-supply spillovers via the final good production, and the extensive margin adjustment of exporters over business cycles. In addition, MP extensive margin and technology transfer between parent and affiliate firms also determine comovement endogenously. Simulations with the model do well to match the

pattern of changes in trade and FDI coefficients across different empirical regressions. The magnitude of the coefficients are sensitive to the volatility of MP extensive margin.

The model has some important shortcomings. Adding dynamic elements to firm decision making, such as adding sunk cost of MP, is an important dimension along which model can be improved. Doing so will facilitate calibrating the model to the volatility of the extensive margin of affiliates. Also, allowing for asymmetry between countries, and current account imbalances are two promising extensions.

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## 7 Appendix

### 7.1 Empirical Appendix

#### A. Data Sources And Description:

For the empirical analysis, I gather data on quarterly and annual real GDP, annual bilateral exports and bilateral stock of FDI. Except for the data on trade, all data comes from OECD database. The list of countries in my analysis matches the one in [de Soyres \[2016\]](#), but the time period is different. In the baseline analysis, I gather data for 1993-2012 and split the period into two non-overlapping ten year intervals.

Data on bilateral exports were downloaded from the UN COMTRADE database. I use total value of all exports at annual frequency. Annual and quarterly real GDP are available from the OECD database (variable name: “VPVOBARSA”). To compute correlation of real GDP, I use quarterly real GDP from OECD. I use quarterly GDP to increase number of datapoints for the filters used to compute cyclical components. I detrend the entire series of data from 1993-2012 before computing correlations for 10 year non-overlapping intervals. Data on FDI stock is also available from the OECD database under the “activity of multinationals” category. While the data is available from 1985 to 2013, the earlier years have many missing values and are not suitable for the analysis. Even during 1993-2012, I consider country pairs that have at least one FDI Instock and Outstock values during both of the 10-year intervals.

Table 6: Top 10 countries

Comovement			Trade Share			FDI Share		
1.	AUT-FRA	0.92	1.	CAN-USA	0.42	1.	NLD-USA	0.36
2.	ESP-NLD	0.91	2.	MEX-USA	0.33	2.	CHE-USA	0.33
3.	CHE-NLD	0.88	3.	GBR-IRL	0.25	3.	IRL-USA	0.32
4.	AUT-NLD	0.88	4.	AUR-DEU	0.25	4.	CAN-USA	0.30
5.	FRA-NLD	0.87	5.	DEU-NLD	0.19	5.	GBR-IRL	0.27
6.	CHE-ESP	0.87	6.	IRL-USA	0.16	6.	GBR-NLD	0.26
7.	DEU-FRA	0.87	7.	CHE-DEU	0.15	7.	GBR-USA	0.24
8.	FIN-SWE	0.87	8.	ESP-PRT	0.12	8.	DEU-NLD	0.19
9.	CHE-FRA	0.86	9.	DEU-DNK	0.10	9.	IRL-NLD	0.17
10.	FRA-SWE	0.86	10.	DEU-IRL	0.098	10.	FIN-SWE	0.16

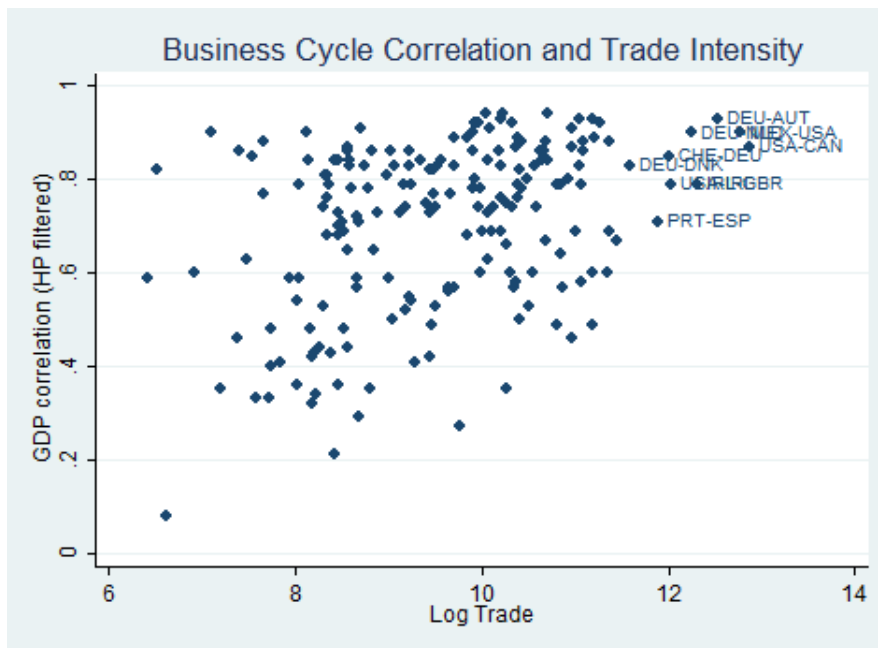
Top 10 country-pairs by comovement, trade share and FDI share. Computed using data from 1993-2012 for 20 OECD countries. Comovement, Trade and FDI shares are as defined in eq. 1.

Table 7: Bottom 10 countries

Comovement			Trade Share ( $10^{-4}$ )			FDI Share ( $10^{-6}$ )		
1.	PRT-TUR	0.12	1.	MEX-TUR	4.6	1.	AUS-TUR	0.0086
2.	AUS-ITA	0.16	2.	MEX-NOR	5.3	2.	AUS-MEX	0.12
3.	AUS-JPN	0.17	3.	AUS-PRT	7.8	3.	MEX-TUR	0.12
4.	AUS-TUR	0.19	4.	DNK-MEX	9.3	4.	AUT-MEX	1.1
5.	DEU-TUR	0.20	5.	AUS-NOR	10.3	5.	PRT-TUR	1.5
6.	ESP-TUR	0.20	6.	AUS-MEX	11.1	6.	AUS-PRT	21.5
7.	AUS-PRT	0.21	7.	AUT-MEX	11.8	7.	CAN-TUR	114.7
8.	AUS-DEU	0.21	8.	AUS-TUR	12.7	8.	MEX-PRT	130.3
9.	NOR-PRT	0.22	9.	FIN-MEX	13.1	9.	FIN-MEX	175.3
10.	NLD-TUR	0.23	10.	MEX-PRT	16.2	10.	MEX-NOR	184.1

Bottom 10 country-pairs by comovement, trade share and FDI share. Computed using data from 1993-2012 for 20 OECD countries. Comovement, Trade and FDI shares are as defined in eq. 1.

### 7.1.1 Trade and GDP Correlation



### 7.1.2 Summary Statistics

Table 8: Summary Statistics

	mean	sd	p50	min	max	count
HP filtered correlation	0.576	0.295	0.630	-0.310	0.950	364
BK filtered correlation	0.607	0.346	0.705	-0.580	0.990	364
Growth rate filtered correlation	0.564	0.289	0.640	-0.320	0.950	364
FDI share (max)	0.039	0.069	0.012	0.000	0.461	364
Trade share (max)	0.032	0.053	0.016	0.000	0.471	364

*Notes:* The table above provides a summary statistics of the variables in empirical analysis.

### 7.1.3 Robustness Checks

Table 9: Results - BK filtered GDP

	(1)	(2)	(3)
Log(Trade)	0.36** (0.12)		0.27* (0.13)
Log(FDI)		0.05** (0.02)	0.04* (0.02)
Country-pair FE	Yes	Yes	Yes
Observations	364	364	364
$R^2$	0.463	0.462	0.475

Standard errors in parentheses

Results from regressing correlations of BK-filtered GDP as the comovement measure.

Sample: 182 country-pairs between 20 OECD economies per time period, and two time periods covering 1993-2012.

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 10: Results using Hodrik-Prescott Filtered GDP

	(1)	(2)	(3)
Log(Trade)	0.385*** (0.092)		0.049 (0.087)
Log(FDI)		0.218*** (0.020)	0.214*** (0.021)
Country-pair FE	Yes	Yes	Yes
Observations	504	504	504
$R^2$	0.353	0.503	0.503

Standard errors in parentheses

Results from estimating for three time periods between 1992-2012.

Sample: 168 country-pairs among 20 OECD countries per time period.

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 11: Results using Hodrik-Prescott Filtered GDP

	(1)	(2)	(3)
Log(Trade)	0.819*** (0.130)		0.453*** (0.128)
Log(FDI)		0.257*** (0.029)	0.209*** (0.031)
Country-pair FE	Yes	Yes	Yes
Observations	336	336	336
$R^2$	0.649	0.702	0.723

Standard errors in parentheses

Results from estimating the baseline equation before the Great Recession.

Sample 168 country-pairs among 20 OECD economies per time period, and two time periods between 1992-2005

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

## 7.2 Theoretical Appendix

### 7.2.1 Model Equations

#### Household FOCs

Labor Supply

$$L_{i,t}^\nu = \frac{w_{i,t}}{C_{i,t}^\psi} \quad (9)$$

Consumption-Investment Euler equation

$$\frac{1}{C_{i,t}} = \beta \mathbb{E}_t \left[ \frac{1}{C_{i,t+1}} (r_{i,t+1} + 1 - \delta) \right] \quad (10)$$

#### Cutoffs

$$\varphi_{ik,t}^X = \frac{\sigma}{\sigma - 1} \left[ \left( \frac{mc_{it}}{Q_{ik}} \right)^\sigma \sigma \frac{F^X}{Y_{k,t}} \right]^{\frac{1}{\sigma-1}} \tau$$

$$\varphi_{ik,t}^M = \frac{\sigma}{\sigma - 1} \left[ \frac{mc_{k,t} h \left( \frac{Z_{k,t}}{Z_{i,t}} \right)^\mu F^M - \frac{mc_{i,t}}{Q_{ik}} F^X}{\left( mc_{k,t} h \left( \frac{Z_{k,t}}{Z_{i,t}} \right)^\mu \right)^{1-\sigma} - \left( \frac{mc_{i,t}}{Q_{ik}} \right)^{1-\sigma}} \frac{\sigma}{Y_{k,t}} \right]^{\frac{1}{\sigma-1}}$$

Where  $Y_{i,t} = C_{i,t} + X_{i,t}$  is real consumption and investment expenditure in country  $i$ .

#### Average Productivity by Activity

$$\tilde{\varphi}_i^D = B$$

$$\tilde{\varphi}_{ik}^X = B \left[ \frac{(\varphi_{ik}^X)^{\sigma-\gamma-1} - (\varphi_{ik}^M)^{\sigma-\gamma-1}}{(\varphi_{ik}^X)^{-\gamma} - (\varphi_{ik}^M)^{-\gamma}} \right]^{\frac{1}{\sigma-1}}$$

$$\tilde{\varphi}_{ik}^M = B \varphi_{ik}^M$$

where  $B = \left( \frac{\gamma}{1+\gamma-\sigma} \right)^{\frac{1}{\sigma-1}}$  is a constant.

**Number of Firms by Activity** Given number of potential entrants  $M_i$  in each country,



number of firms that take up different activities is given by,

$$M_{ik}^X = (G(\varphi_{ik}^M) - G(\varphi_{ik}^X))M_i = ((\varphi_{ik}^X)^{-\gamma} - (\varphi_{ik}^M)^{-\gamma})M_i$$

$$M_{ik}^M = (1 - G(\varphi_{ik}^M))M_i = (\varphi_{ik}^M)^{-\gamma}M_i$$

## Aggregate Prices

### Price of Consumption Bundle

$$1 = M_{i,t}^D (\rho^D(\tilde{\varphi}_{it}^D))^{1-\sigma} + \sum_{k \neq i} M_{ki,t}^X (\rho_{ki,t}^X(\tilde{\varphi}_{ki,t}^X))^{1-\sigma} + \sum_{k \neq i} M_{ki,t}^M (\rho_{ki,t}^M(\tilde{\varphi}_{ki,t}^M))^{1-\sigma} \quad (11)$$

### Domestic Price Index

$$P_{i,t}^D = (M_i^D)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{P_{it} m c_{it}}{\tilde{\varphi}_i^D}$$

### Export Price Index

$$P_{ik,t}^X = (M_{ik}^X)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{P_{it} m c_{it} \tau}{\tilde{\varphi}_{ik}^x}$$

### MP Price Index

$$P_{ik,t}^M = (M_{ik}^M)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{P_{k,t} m c_{k,t} h}{\tilde{\varphi}_{ik}^M} \left( \frac{Z_{k,t}}{Z_{i,t}} \right)^\mu$$

## Steady State

Model is said to be in steady state when the aggregate productivities are at their long run expected values in all the countries and as a result, the endogenous variables are time invariant. In an N-country model there are 11N unknowns (TFP, wages, rental rates for capital, prices, number of entrants, consumption, investment, labor supply, stock of capital, 3N cutoffs) which are solved for such that,

1. Agregate TFPs are in long run expected values in all countries
2. Given wages, rental rates and prices, domestic and export cutoffs satisfy ZCP conditions and Multinational cutoff satisfies MP entry condition (see appendix for equations)
3. Given rental rates, Euler equations determine aggregate prices

4. Investments are a constant fraction  $\delta$  of aggregate capital stock
5. Given household income, stock of capital and aggregate prices, goods market clearing condition in (7) determines consumption
6. Given wages, prices and cutoffs, free entry condition determines number of entrants
7. Given wages, prices, cutoffs and number of entrants, labor demands implied by (9) equal household labor supply.
8. Rental rates satisfy capital market equilibrium conditions in (8).
9. N-1 current account balance in (5) conditions determine wages.

Above conditions provide  $11N-1$  equations, while there are  $11N$  unknowns. Normalizing wage in one of the countries to 1 provides the final equation to solve for the steady state.

## 7.2.2 Theoretical Proofs

### Proof of Lemma 1

$$\begin{aligned}
& \varphi_{ik,t}^M > \varphi_{ik,t}^X \\
\Rightarrow & \frac{\sigma}{\sigma-1} \left[ \frac{mc_{k,t} h_{ik} \left(\frac{Z_k}{Z_i}\right)^\mu F_{ik}^M - \frac{mc_{i,t}}{Q_{ik}} F_{ik}^X}{\left(mc_{k,t} h_{ik} \left(\frac{Z_k}{Z_i}\right)^\mu\right)^{1-\sigma} - \left(\frac{mc_{i,t} \tau_{ik}}{Q_{ik}}\right)^{1-\sigma} Y_{k,t}} \sigma \right]^{\frac{1}{\sigma-1}} > \frac{\sigma}{\sigma-1} \left[ \left(\frac{mc_{it}}{Q_{ik}}\right)^\sigma \sigma \frac{F_{ik}^X}{Y_{k,t}} \right]^{\frac{1}{\sigma-1}} \tau_{ik} \\
& \Rightarrow \frac{mc_{k,t} h_{ik} \left(\frac{Z_k}{Z_i}\right)^\mu F_{ik}^M - \frac{mc_{i,t}}{Q_{ik}} F_{ik}^X}{\left(mc_{k,t} h_{ik} \left(\frac{Z_k}{Z_i}\right)^\mu\right)^{1-\sigma} - \left(\frac{mc_{i,t} \tau_{ik}}{Q_{ik}}\right)^{1-\sigma}} > \tau_{ik}^{\sigma-1} \left(\frac{mc_{it}}{Q_{ik,t}}\right)^\sigma F_{ik}^X \\
& \Rightarrow F_{ik}^M > F_{ik}^X \tau_{ik}^{\sigma-1} \left(\frac{mc_{it} h_{ik}}{Q_{ik,t} mc_{kt}} \left(\frac{Z_{it}}{Z_{kt}}\right)^\mu\right)^\sigma \\
& \Rightarrow \frac{F_{ik}^M h_{ik}^\sigma}{F_{ik}^X \tau_{ik}^{\sigma-1}} > \left[ \frac{mc_{it}}{Q_{ik,t} mc_{kt}} \left(\frac{Z_{it}}{Z_{kt}}\right)^\mu \right]^\sigma
\end{aligned}$$

Q.E.D

### 7.2.3 Theory - Other Graphs

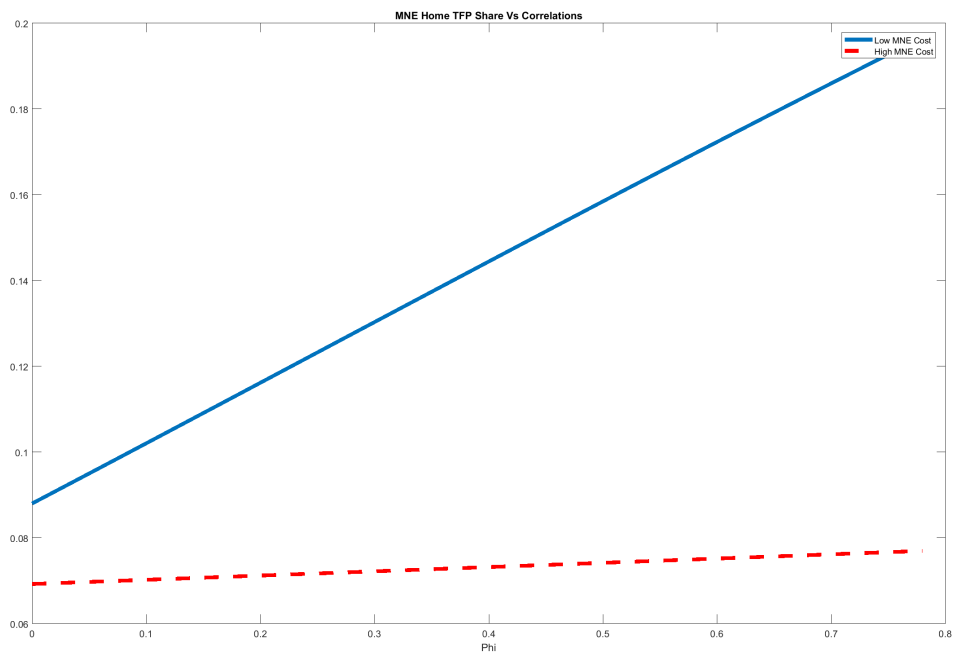
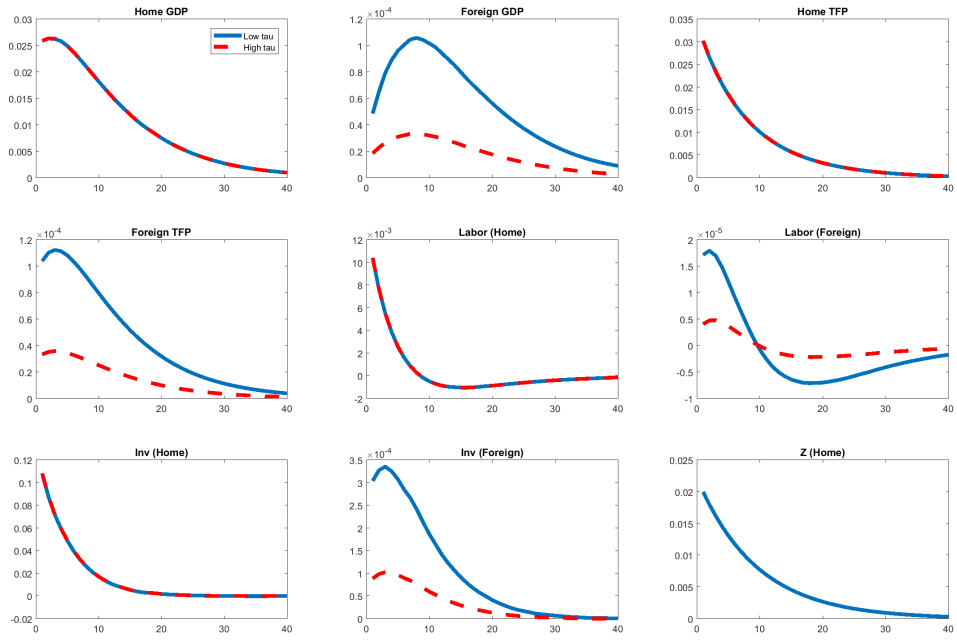
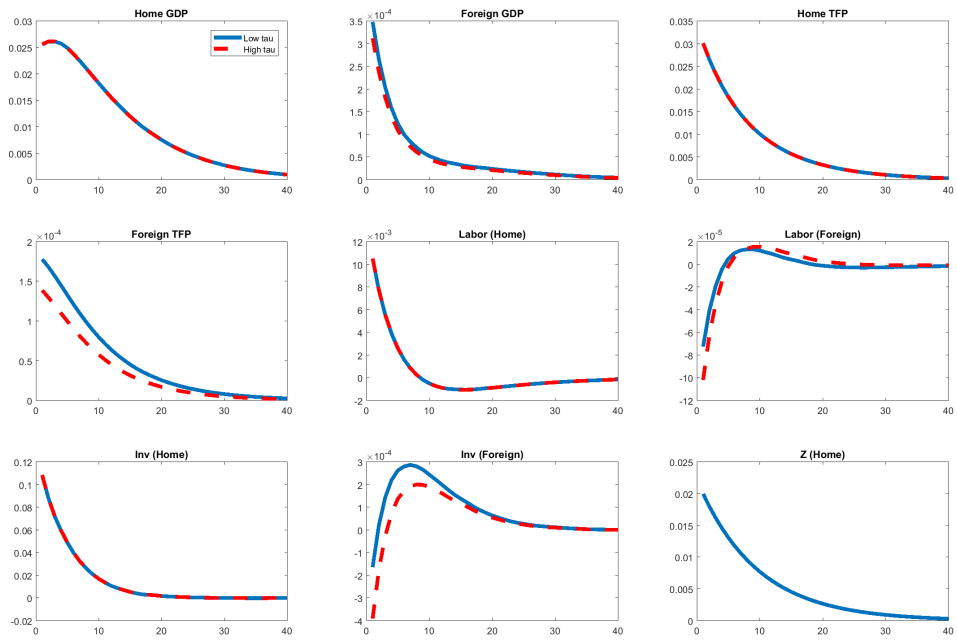


Figure 4: Correlation of real GDP under different values of  $\mu$  under baseline calibration.



(a) Varying  $\tau$  under prohibitively high MP costs.



(b) Varying  $\tau$  under low MP costs.

### 7.3 Quantitative Exercises

#### Data and Methodology

In the quantitative exercises, I recalibrate my model, holding all parameters except the fixed and iceberg costs of trade and MP, to match different combinations of trade and FDI shares. The relationship between them in my sample is,

$$\text{Trade Sh} = 0.00129 + 0.0599 \text{ FDI Sh} + \epsilon_t \quad \text{Std.Err}(\epsilon_t) = 0.0056791$$

I start by selecting 50 equally spaced FDI shares between 10% and 30%, and repeat each of these shares 4-times. This gives me a 200-by-1 vector of FDI shares. I generate trade shares using the relationship from the data. This generates variation in trade shares for each value of FDI share. When generating the trade shares, I scale the intercept coefficient to be able to match parameters.

The resulting distributions of the moments are presented below,

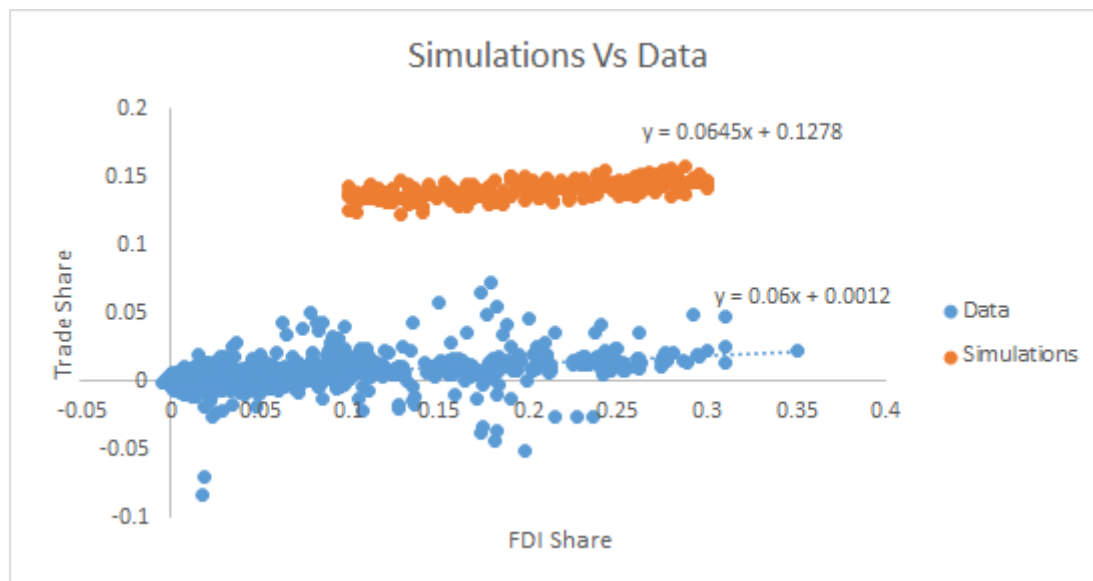


Figure 6: Comparison of trade and FDI shares in data and quantitative exercise. Trade share was plotted after removing country-pair fixed effects.

Table 12: Quantitative Exercises - Robustness

	$\sigma = 4$					
	$\mu = 0$			$\mu = 0.3$		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$\beta_{trade}$	0.28*** (0.02)		0.09*** (0.00)	1.00*** (0.09)		0.36*** (0.02)
$\beta_{FDI}$		0.06*** (0.00)	0.05*** (0.00)		0.24*** (0.00)	0.21*** (0.00)
	$\sigma = 3$					
	$\mu = 0$			$\mu = 0.3$		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
$\beta_{trade}$	0.28*** (0.02)		0.08*** (0.00)	1.06*** (0.09)		0.27*** (0.01)
$\beta_{FDI}$		0.06*** (0.00)	0.05*** (0.00)		0.25*** (0.00)	0.22*** (0.00)

Results from estimating trade-comovement regression with model implied numbers for 200 artificial country-pairs.

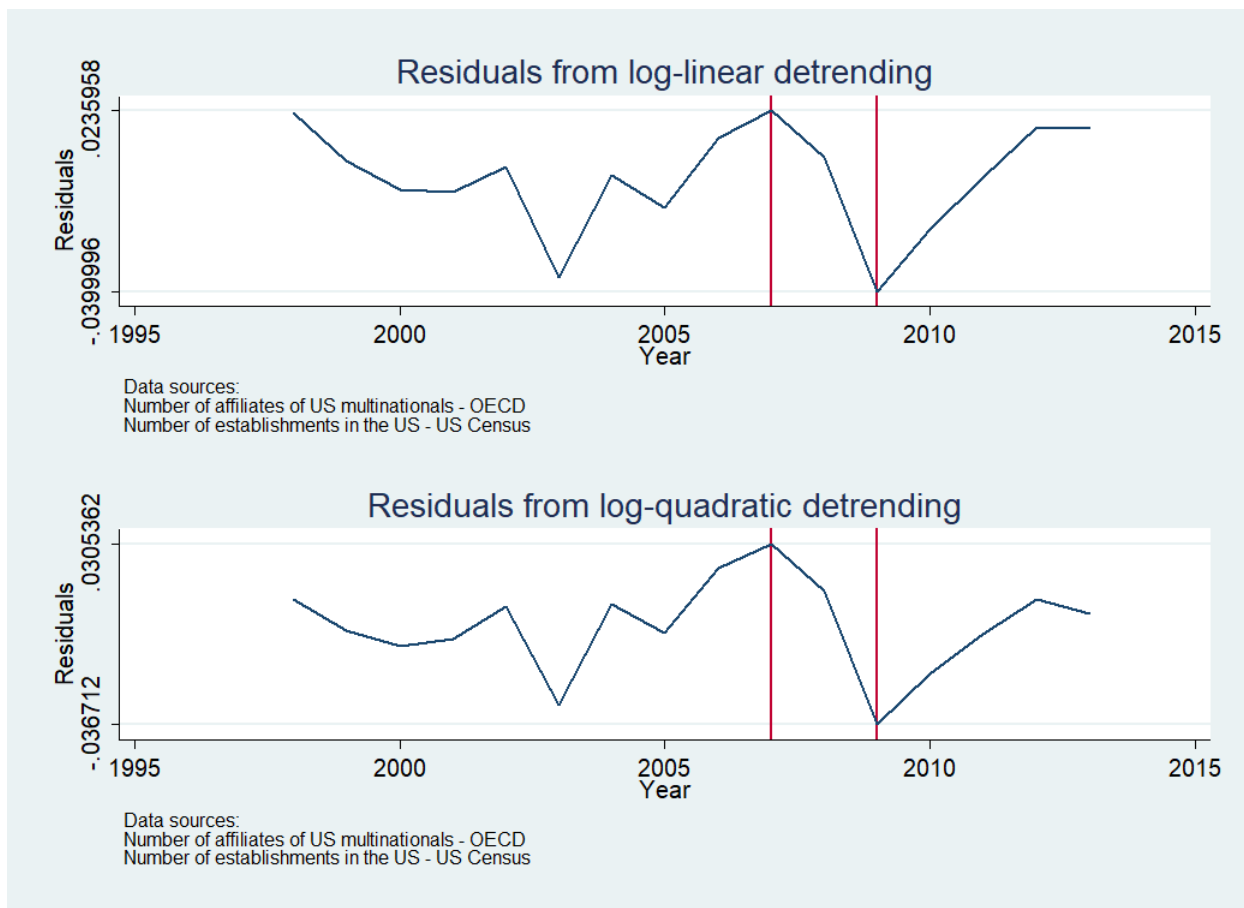


Figure 7: Residuals from log-linear and log-quadratic detrending of the affiliate share. Affiliate share is defined as the total number of all US owned affiliates located in RoW, divided by the total number of US manufacturing establishments.