# International Business Cycles- The Role of Technology and Resource Transfers Within Multinationals

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**Abstract**

Recent empirical research has identified mechanisms through which multinational firms affect international business cycle comovement. To quantify these mechanisms, I develop a general equilibrium model of trade and multinational production (MP) with firm heterogeneity, market access frictions, and physical capital. Because firms own physical capital, their entry and exit into MP generates transfer of this productive resource across countries. I focus on the interaction of this novel channel with the within-firm technology transfer channel and quantify its implications for international output comovement. Resource transfer channel worsens comovement as multinationals move capital towards the high growth country, but technology transfer augments comovement. When calibrated to the United States, output comovement is highest in the full model, followed by a no trade model, and least in an MP model with only the resource transfer channel. Of the increase in comovement between no-trade and full model, 30% is due to MP entry and exit.

**JEL Classifications**: F21, F23, F44.
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# 1 Introduction

Is multinational production (MP), where firms produce in more than one country, an important avenue for business cycle synchronization across countries? The last three decades have seen dramatic increases in both cross-country output comovement and sales of affiliates of multinational firms as a fraction of global gross domestic product (GDP), but theoretical research that puts the two together remains limited. Instead, theoretical research has evaluated cross-border trade as a possible spillover channel but most quantitative trade models have failed to generate the increase in output correlations observed in the data.[1]

Even within the literature on multinationals and international business cycles, very few papers have built quantitative models that are comparable to existing trade models.[2] The focus instead has been empirical, thanks to availability of detailed multi-country firm level datasets with information on global ultimate ownership.[3] These papers have documented strong micro-correlations between multinational parents and affiliates sales. Building on these micro-correlations, they however conclude that at the aggregate trade linkages quantitatively dominate MP linkages primarily because bilateral MP shares are much smaller than bilateral trade shares. In this paper, I argue that the empirical research misses formation and destruction of trade and MP linkages (i.e., their extensive margins) which have implications for international business cycles. Moreover, thanks to their large market share, MP firms' entry and exit have as large quantitative effects as that of exporters in spite of there being much fewer MP firms compared to exporters.

I quantify these effects in this paper by extending a standard heterogeneous firm model of trade and market access friction to incorporate multinational production. As in Alessandria and Choi [2007], firms differ by their productivity, own their physical capital stock, and are subject to aggregate and idiosyncratic shocks to their total factor productivity (TFP). Market access frictions, modeled as fixed export cost and fixed and sunk MP costs, reduce firms' profits from engaging in these activities. This results in a segregation of firms into domestic, export, and MP based on their idiosyncratic productivities and last-period MP status. On average, in the cross section, the most productive firms conduct MP, firms with intermediate productivities export, and the least productive firms only serve the domestic market. Over time, firms' export and MP statuses vary with the aggregate and the idiosyncratic states. In addition to this cross sectional separation, the

[1] For empirical trade-comovement literature, see Frankel and Rose [1998] and Imbs [2004] among others; for quantitative work see Liao and Santacreu [2015], Johnson [2014], de Soyres [2016], Kose and Yi [2006] and citations therein.

[2] Contessi [2010] and Zlate [2016] are two exceptions. Contessi [2010] models heterogeneous firms with horizontal MP in the absence of resource and technology transfers. Zlate [2016] models north-south vertical multinationals motivated by the United States-Mexican maquiladora relationship.

[3] See di Giovanni et al. [2018, 2017], di Giovanni and Levchenko [2010], Cravino and Levchenko [2017] and others.

sunk MP cost adds persistence to firms' MP status. The choice between exporting and MP is therefore driven by the sunk and fixed costs, aggregate states in the two countries (wages and TFP), and the level of technology transfer (described below). To summarize, I develop a horizontal MP model that includes capital and generates hysteresis in MP status.[4]

The main focus of the paper is the interaction between within-firm technology and resource transfer channels and their effect on aggregate output comovement. I assume that a constant fraction of affiliates' aggregate productivity comes from their parents, while the the idiosyncratic component is transferred fully.[5] Each new affiliate therefore brings in source country's aggregate technology, its idiosyncratic technology, and a stock of capital to start its operation in the host country.

Technology transfer plays an important role in this MP model. Consider a positive aggregate productivity shock in one country (Home). This results in a fall in relative effective wage rate (i.e., terms of labor) in Home so production in Home is cheaper. In response, firms' incentives to engage in trade versus MP is altered by how much of the parent aggregate productivity they get to take abroad. Higher the level of this technology transfer, the better it is for Home firms to conduct MP and Foreign (the other country) firms to export. This is because Home firms get to enjoy higher affiliate productivity to compensate for the loss in terms of labor. At the same time, Foreign MP firms are disadvantaged by carrying a lower parent aggregate productivity. As a result, there is less net inflow of MP firms towards Home with higher technology transfer. This means that the Foreign output does not fall as much compared to the no technology transfer case (described below), which generates higher comovement.

Ignoring technology transfer reduces output comovement significantly (results from calibrated model is below). The mechanisms driving comovement in the absence of technology transfer are as follows. First, the shock results in a larger net inflow of affiliates and capital at the expense Foreign. With greater number of firms exiting production in Foreign, its output deviates further from Home. Given everything else, this reduces comovement. This channel is weakened with higher level of technology transfer, as described above. Second, there is an increase in number of both Home (because Home products are

[4]There is substantial evidence that much of the multinational activity between developed economies is of horizontal type where affiliates produce the same good as their parent to sell in the host market with little or no trade with the parent firm. Ramondo et al. [2016] report that the median foreign affiliate of United States' multinational corporations sells 91% of its output to unaffiliated parties, of which 73% goes to unaffiliated parties in the country of affiliates operation. The distribution is highly skewed - only a small group of firms in the 95th percentile exclusively trade with affiliated parties. On the other side of the distribution, 50% of the affiliates ship nothing to - and receive nothing from - their parent entities.

[5]This formulation combines Cravino and Levchenko [2017] and Contessi [2010]. Cravino and Levchenko [2017] model affiliate TFP as a combination of its parent and affiliate idiosyncratic and aggregate components. Contessi [2010] assumes that affiliate TFP is a product of host aggregate productivity and parent idiosyncratic productivity. So idiosyncratic productivity is transferred fully but aggregate productivity is not.

cheaper) and Foreign exporters (a demand spillover effect). While the latter contributes to foreign GDP directly, the former also has a positive variety effect, as shown in Liao and Santacreu [2015]. Third, because there is risk sharing via internationally traded bonds, Home runs a trade deficit as its firms invest more. Output comovement falls as Home accumulates more capital over time.

To quantify these mechanisms, I calibrate the model to match extensive and intensive margins of trade and MP for the United States (US). I set the technology transfer level to a value estimated in Cravino and Levchenko [2017]. They estimate a value between 20-40%, so I take a mid-point of 30% in the benchmark simulations. An MP model with technology transfer parameter set to zero (to test the importance of technology transfer), and a trade-only model are used for comparison. Output comovement in benchmark model is higher by 13 percentage points at 0.29 compared to no MP model which has a comovement measure of 0.19. The results are dependent on technology transfer to the extent that in the zero technology transfer model, output comovement at 0.08 is lower than the trade-only model.

It is relevant to compare the within-firm resource transfer mechanism explained above with the resource transfer mechanism in existing trade models due to international bond markets (i.e., mechanism three above). The resource transfer mechanism in trade-only models is gradual and is a result of household investment decisions: Home households borrow internationally to invest more while the Foreign households lower their investments, and the two countries' stock of capital diverge over time. This too reduces output comovement, as pointed out in Kose and Yi [2006]. Under the zero technology transfer calibration in this paper, the traditional resource transfer channel is further strengthened by multinationals as transfer of capital stock via multinationals is instantaneous.

An important contribution of this paper is to bring together different elements of MP, including technology transfer and capital, into a single model. Its motivation is similar to Contessi [2010] in looking at the effect of horizontal multinational activity on international business cycle moments. This paper however differs in three important ways. First, there is capital which improves model performance by matching variation in net-exports better with the data. In particular, net exports fall in response to an aggregate productivity shock unlike Contessi [2010] where net exports increase. Second, because labor supply is elastic, this model also matches evolution of terms of labor (exchange rate adjusted real effective wage differential across countries) and real exchange rates across countries better. Existing literature shows that an aggregate productivity shock reduces prices and depreciates real exchange rate. However, because labor supply is inelastic in Contessi [2010], home wage increases more (relative to foreign) and the prices increase. This results in an appreciation of the real exchange rate. Third, I include sunk cost of engaging in multinational production. This results in hysteresis in firms' MP status as firms that engaged in MP last period are more likely to continue in spite of a bad current idiosyncratic productivity. This feature helps to discipline firms' entry and exit into MP better.

In contrast to this paper and Contessi [2010], Zlate [2016] looks at the effect of north-south type of vertically fragmented MP and its effects on international business cycles. His paper is motivated by the US-Mexican maquiladora relationship where US multinationals produce in Mexico, but the affiliate output is shipped back to the US. This type of MP boosts output complement across countries because there is a direct spillover of US demand on production in the maquiladoras. This paper focuses on the effects of the more dominant type of MP between high-income countries (i.e, of the north-north type) and its effect on international business cycles.

This paper contributes to the expanding literature on business cycle comovement arising from multinational activity (see Boehm et al. [2019b], Budd et al. [2005], Buch and Lipponer [2005], Burstein et al. [2008], Cravino and Levchenko [2017], Contessi [2010], Desai et al. [2009], Desai and Foley [2006], di Giovanni et al. [2018], Kleinert et al. [2015], and Zlate [2016]). It emphasises the role played by parent to affiliate technology transfer as a way to reconcile two features of multinational activity and international business cycle comovement in the data: while business cycles are more synchronized in the last thirty years, increase in vertical fragmentation of production, as modeled in Burstein et al. [2008] and Zlate [2016] for the case of United States and Mexican maquiladoras, have been shown to not be a major contributor in a larger global context. Compared to these papers, the main point of departure here is to model horizontal multinational activity.

The rest of the paper is organized as follows- section 2 proposes a business cycle model with heterogeneous firms, trade, and horizontal MP; section 3 details the calibration procedure; section 4 provides the results and intuition, and section 5 concludes.

# 2 Model

In this section, I develop a model of trade and horizontal MP with firm heterogeneity and market access frictions. There are two countries denoted by Home and Foreign. Within each country and each period, there are intermediate good producers of unit measure. Infinitely lived representative household chooses how much to consume and work and to save in an array of state contingent internationally traded bonds. Shocks in a particular period $t$ are encapsulated in the term $s_t$, while the history of shocks until that period are given by the set $s^t = (s_0, s_1, ..., s_t)$.

## 2.1 Households

I follow the Alessandria and Choi [2007] notation closely and denote variables corresponding to Foreign with an asterisk. Households in Home maximize expected discounted lifetime utility by choosing consumption, labor supply, and bond holdings.

$$U(s_0) = \max_{C(s^t),L(s^t),B(s^t)} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0) \frac{[C(s^t)^\gamma (1-L(s^t))^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

where $\beta$ is the discount factor, $\pi(s^t|s_0)$ is the conditional probability of $s^t$ given $s_0$, $C(s^t)$ is Home consumption, $L(s^t)$ is Home labor supply, and $\gamma$ and $\sigma$ are consumption share in composite commodity and the intertemporal elasticity respectively. The intertemporal budget constraint for this Home household is given by,

$$P(s^t)C(s^t) + \sum_{s^{t+1}} Q(s^{s+1}|s^t)B(s^{t+1}) = P(s^t)W(s^t)L(s^t) + B(s^t) + \Pi(s^t)$$

where $P(s^t)$ is the price of aggregate final good, $Q(s^{t+1})$ is the price of state-contingent bond $B(s^{t+1})$ at time t, $W(s^t)$ is the real wage, and $\Pi(s^t)$ is the total profits of intermediate good produces owned by Home households. The Foreign utility function and the budget constraint are defined similarly using $e(s^t)$ as the nominal exchange rate between the two countries. The Foreign household's budget constraint is,

$$P^*(s^t)C^*(s^t) + \sum_{s^{t+1}} \frac{Q(s^{s+1}|s^t)}{e(s^t)} B^*(s^{t+1}) = P^*(s^t)W^*(s^t)L^*(s^t) + \frac{B^*(s^t)}{e(s^t)} + \Pi^*(s^t)$$

The first-order conditions for the home household are,

$$\frac{U_L(s^t)}{U_C(s^t)} = W(s^t) \tag{1}$$

$$Q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t) \frac{U_C(s^{t+1})}{U_C(s^t)} \frac{P(s^t)}{P(s^{t+1})} \tag{2}$$

where $W(s^t)$ is the real wage rate in Home and $Q(s^{t+1}|s^t)$ is the stochastic discount factor.

## 2.2 Final good producer

The Home final good producer combines all the varieties available in Home. The production function is given by,

$$D(s^t) = \left[ \delta_h \left[ \int_{i=0}^{1} y_h(i,s^t)^\theta di \right]^{\frac{\rho}{\theta}} + \delta_x \left[ \int_{\xi^{X*}(s^t)} y_{xf}(i,s^t)^\theta di \right]^{\frac{\rho}{\theta}} + (1-\delta_h-\delta_x) \left[ \int_{\xi^{M*}(s^t)} y_{mf}(i,s^t)^\theta di \right]^{\frac{\rho}{\theta}} \right]^{\frac{1}{\rho}}$$

where $D(s^t)$ the output of the final good, $y_h(i, s^t)$ is the intermediate Home variety sold domestically, $y_{xf}(i, s^t)$ is the intermediate variety exported from Foreign, and $y_{mf}(i, s^t)$ is the intermediate from Foreign multinationals produced at Home; $\delta_h$ and $\delta_x$ are home bias and import share parameters; $\xi^{X*}(s^t)$ and $\xi^{M*}(s^t)$ are respectively the set of varieties exported and served by multinationals from Foreign. Note that both $\xi^{X*}(s^t)$ and $\xi^{F*}(s^t)$ are sets of Foreign varieties sold in Home, but the production location is different for the two sets. Given the production function above, the elasticity of substitution between two varieties from the same country is $\frac{1}{1-\theta}$; the elasticity between varieties from different countries is $\frac{1}{1-\rho}$.

Let $P_h(i, s^t)$, $P_h(s^t)$, $P_{xf}(s^t)$, and $P_{mf}(s^t)$ be the Home price of a Home-produced variety $i$, the price of the aggregated Home varieties, the price of the aggregated Foreign exported varieties sold in Home, and the price of the aggregated Foreign multinational varieties sold in Home respectively.[6] Then the demand for a given variety from the Home final good producer is,

$$y_h(i, s^t) = \delta_h^{\frac{1}{1-\rho}} \left[\frac{P_h(i, s^t)}{P(s^t)}\right]^{\frac{1}{\theta-1}} \left[\frac{P_h(s^t)}{P(s^t)}\right]^{\mu} D(s^t) \tag{3}$$

$$y_{xf}(i, s^t) = \delta_x^{\frac{1}{1-\rho}} \left[\frac{P_{xf}(i, s^t)}{P(s^t)}\right]^{\frac{1}{\theta-1}} \left[\frac{P_{xf}(s^t)}{P(s^t)}\right]^{\mu} D(s^t) \tag{4}$$

$$y_{mf}(i, s^t) = (1 - \delta_h - \delta_x)^{\frac{1}{1-\rho}} \left[\frac{P_{mf}(i, s^t)}{P(s^t)}\right]^{\frac{1}{\theta-1}} \left[\frac{P_{mf}(s^t)}{P(s^t)}\right]^{\mu} D(s^t) \tag{5}$$

where $\mu = 1/(1-\theta) - 1/(1-\rho)$ is the difference in elasticities between domestic and foreign aggregates. Since exporting involves an iceberg cost, an exporter's price is higher compared to the case when it were to set up an affiliate. Given everything else, this implies that a firm faces higher demand if it were to conduct MP. Finally, the aggregate price in Home is a combination of domestic, imported, and foreign affiliate prices,

$$P(s^t) = \left[\delta_h^{\frac{1}{1-\rho}} P_h(s^t)^{\frac{\rho}{\rho-1}} + \delta_x^{\frac{1}{1-\rho}} P_{xf}(s^t)^{\frac{\rho}{\rho-1}} + (1 - \delta_h - \delta_x)^{\frac{1}{1-\rho}} P_{mf}(s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{6}$$

[6] $P_h(s^t) = \left[\int_0^1 P_h(i, s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$, $P_{xf}(s^t) = \left[\int_{\xi^{X*}(s^t)} P_{xf}(i, s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$, $P_{mf}(s^t) = \left[\int_{\xi^{M*}(s^t)} P_{mf}(i, s^t)^{\frac{\theta}{\theta-1}} di\right]^{\frac{\theta-1}{\theta}}$

## 2.3 Intermediate producers

### 2.3.1 Production function

The production function by mode for the intermediate varieties is a Cobb-Douglas combination of appropriate capital & labor, and a TFP,

$$y_{hh}^{D}(i,s^t) = A_{hh}(i,s^t)K_h^D(i,s^t)^{\alpha}L_h^D(i,s^t)^{1-\alpha}$$

$$y_{hf}^{X}(i,s^t) = A_{hh}(i,s^t)K_{hf}^X(i,s^t)^{\alpha}L_{hf}^X(i,s^t)^{1-\alpha}$$

$$y_{hf}^{F}(i,s^t) = A_{hf}(i,s^t)K_{hf}^F(i,s^t)^{\alpha}L_{hf}^F(i,s^t)^{1-\alpha}$$

where $y_{jk}^{a}(i,s^t)$ is the total output in mode $a \in \{domestic(D), export(X), MP(F)\}$, for a firm that originated in country $j$ and producing in $k$ where $j$, $k \in \{h,f\}$; $K_{jk}^a(i,s^{t-1})$ and $L_{jk}^a(i,s^{t-1})$ are the capital and labor defined similarly at the activity-production location level.

Production then must occur subject to the following capital constraint:

$$K_h^D(i,s^t) + K_{hf}^X(i,s^t) + K_{hf}^F(i,s^t) \leq K(i,s^{t-1}) \tag{7}$$

The objects on the left hand side of the above equation are denoted in the current time, while the right hand side is in the last-period. This is simply to emphasize the fact that a firm can choose today the capital it allocates to different activities (objects in the lhs), but the capital stock itself was carried over from the previous period (the rhs).

The interaction of firm-owned capital and multinational production generates interesting features that are focus of this paper. Equation 7 assumes that capital market must clear within each firm: if a firm then decides to be a multinational, its capital must come from its parent entity and the sum of the parent's and affiliate's capital must in equilibrium equal the total capital a firm is born with. If on the other hand a firm decides not to conduct MP, its problem is identical to that in firm-owned capital models without MP, such as Alessandria and Choi [2007]. This way of modelling firm owned capital and MP mimics the idea that multinationals have internal capital markets, which I extend to a business cycle context.[7,8]

[7] Existing corporate finance literature points to two reasons why internal capital markets are optimal for multinational firms: i. to move investments from lagging production units to more productive ones, i.e, conduct "winner-picking", which improves the global diversification premium they can offer to their investors (Stein [1997], Sturgess [2016]), and ii. to reduce dependence on external financing when financial markets and underdeveloped and institutions are weak.

[8] Alternatively, one can think of the firm capital to be an amalgam of tangible and intangible components and technology capital. McGrattan and Prescott [2009], for example, model these components explicitly, where the main feature is that technology capital is firm specific but can be used in multiple locations. While I do not model technology capital and its accumulation, thinking of firm capital as an amalgamation of tangible and intangible sub-components helps in rationalising how capital moves across countries at business cycle frequencies in the model.

The MP technology transfer is a combination of Cravino and Levchenko [2017], Zlate [2016], and Contessi [2010]. Cravino and Levchenko [2017] model affiliate productivity as a combination of aggregate and idiosyncratic components of parent and affiliate entities. For computational simplicity, I assume that idiosyncratic productivity is transferred fully across borders as in Zlate [2016] and Contessi [2010]. The productivity term therefore $A_{jk}(i, s^t)$ involves home and host aggregate and idiosyncratic components,

$$A_{jk}(i, s^t) = \exp\left(\zeta Z_j(s^t) + (1-\zeta)Z_k(s^t) + \eta(i, s^t)\right) \tag{8}$$

where $Z_j(s^t)$ and $Z_k(s^t)$ are home- and host-specific aggregate productivities respectively, $\eta(i, s^t)$ is the firm's idiosyncratic productivity, and $\zeta$ is the technology share parameter such that a fraction $\zeta$ of parents' productivity spills over to the affiliates.

The aggregate productivities follow a vector autoregressive process. In the matrix form,

$$Z(s^t) = MZ(s^{t-1}) + \nu(s^t), \quad \nu(s^t) \overset{iid}{\sim} N(0, \Omega)$$

Where $M$ is the matrix of AR1 parameters, and $\nu$ is the innovation to aggregate productivity, assumed to be iid across countries and over time. The firm specific productivity, $\eta$, is also distributed iid across firms and over time $\eta \overset{iid}{\sim} N(0, \sigma_\eta^2)$.

### 2.3.2 Production costs

Because there are no additional frictions to serve the domestic market, all intermediate firms choose to serve domestically. However, firms face frictions if they wish to serve the foreign market. If a firm chooses to export, it pays an exporting fixed cost $F^X$. On the other hand, if a firm chooses to setup an affiliate abroad and serve locally, it pays a (larger) fixed cost $F_1^F$. In addition, a new affiliate (i.e., a firm which did not have affiliates last period) pays a sunk cost $F_0^F$. I assume that all the fixed costs and the MP sunk cost are paid in labor units in the market being served.

### 2.3.3 Firm value

Total value of a firm $i$ originating in Home is the sum of its discounted expected profits across all activities. For a given period, firms' state variables are: 1. idiosyncratic productivity ($\eta$), 2. capital stock of the firm, 3. MP choice last period, and 4. the aggregate macroeconomic conditions. Firms choose the markets to serve, mode of serving the foreign market conditional on the foreign market being served, optimal allocation of capital across production units, quantity sold in each market, and the level of investment.

In the recursive form, firms' problem can be written as,

$$\begin{aligned}V(\eta, K, m^F, s^t) = \max \Pi^D(i, s^t) + m^{X'}(i, s^t)\Pi^X(i, s^t) + m^{F'}(i, s^t)\Pi^F(i, s^t) \\ - P(s^t)x(i, s^t) + \sum_{s^{t+1}}\sum_{\eta'} Q(s^{t+1}|s^t)Pr(\eta')V(\eta', K', m^{F'}, s^{t+1})\end{aligned} \quad (9)$$

where $\eta$ is firm's idiosyncratic productivity, $m^{X'}(i, s^t)$ and $m^{F'}(i, s^t)$ are indicators that equal 1 if a firm $i$ exports or conducts MP in the current period, $x(i, s^t)$ denotes investment. For every firm, the capital accumulation equation is satisfied, and the capital equilibrium condition holds:

$$(1 - \delta_k)K(i, s^{t-1}) + x(i, s^t) = K(i, s^t) \quad (10)$$

$$K^D(i, s^t) + m^{X'}(i, s^t)K^X(i, s^t) + m^{F'}(i, s^t)K^F(i, s^t) = K(i, s^{t-1}) \quad (11)$$

The domestic, export, and MP profits, in terms of the Home currency are,

$$\Pi^D(i, s^t) = P_h(i, s^t)y^D(i, s^t) - P(s^t)W(s^t)L^D(i, s^t) \quad (12)$$

$$\Pi^X(i, s^t) = e(s^t)P_h^{X\star}(i, s^t)y^X(i, s^t) - P(s^t)W(s^t)L^X(i, s^t) - \frac{e(s^t)P^\star(s^t)W^\star(s^t)}{\exp\left(Z^\star(s^t)\right)}F^X \quad (13)$$

$$\begin{aligned}\Pi^F(i, s^t) = e(s^t)\Big[P_h^{F\star}(i, s^t)y^F(i, s^t) - P^*(s^t)W^*(s^t)L^F(i, s^t) \\ - \frac{P^*(s^t)W^*(s^t)}{\exp\left(\zeta Z(s^t) + (1-\zeta)Z^\star(s^t)\right)}F_1^F - (1 - m^F(i, s^t))\frac{P^*(s^t)W^*(s^t)}{\exp\left(\zeta Z(s^t) + (1-\zeta)Z^\star(s^t)\right)}F_0^F\Big]\end{aligned} \quad (14)$$

### 2.3.4 Productivity cutoffs and choices

Every firm produces in the domestic market, but the set of exporters and affiliates is endogenous. I will denote $\eta_0^X(s^t)$, $\eta_1^X(s^t)$, $\eta_0^F(s^t)$, and $\eta_1^F(s^t)$ as the export and MP productivity thresholds among last-period non-MP and MP firms respectively. I will denote by $V^D(i, s^t)$, $V^X(i, s^t)$, and $V^F(i, s^t)$ the values of a given firm $i$ given its state variables by choosing to serve only the domestic market, being exporter, and conducting MP respectively. Formally, I can define the marginal exporter, given MP status last-period, as the firm for which the following holds:

$$V^D(\eta_{m^F}^X, K, m^F, s^t) = V^X(\eta_{m^F}^X, K, m^F, s^t) \quad (15)$$

And for the marginal MP firm, given MP status last period,

$$V^X(\eta_{m^F}^F, K, m^F, s^t) = V^F(\eta_{m^F}^F, K, m^F, s^t) \quad (16)$$

### 2.3.5 Aggregation

Given the cutoffs, I can write the laws of motion for number of exporters and MP firms originating in each country, $N^X(s^t)$ and $N^F(s^t)$,

$$N^F(s^t) = [1 - \Phi(\eta_1^F(s^t))]N^F(s^{t-1}) + [1 - \Phi(\eta_0^F(s^t))][1 - N^F(s^{t-1})]$$

$$N^X(s^t) = \left[\Phi(\eta_1^F(s^t)) - \Phi(\eta_1^X(s^t))\right] N^F(s^{t-1}) + \left[\Phi(\eta_0^F(s^t)) - \Phi(\eta_0^X(s^t))\right] \left[1 - N^F(s^{t-1})\right]$$

where $\Phi(\cdot)$ is the cumulative density function of $\eta$. Because the firms' idiosyncratic distribution is assumed to be i.i.d., firms' expectation of the future profits are entirely determined by their MP status in the current period. Consequently, every firm that conducts MP today expects the same profit tomorrow, and decides on the same level of capital stock for tomorrow (denoted $K_1(s^t)$); every current non-MP firm also has the same level of capital stock (denoted $K_0(s^t)$). The aggregate capital stock and investment can then be written as,

$$K(s^t) = \left[1 - N(s^t)\right] K_0(s^t) + N(s^t)K_1(s^t)$$

$$X(s^t) = K(s^t) - (1 - \delta_k)K(s^{t-1})$$

The aggregates for labor demand, price indexes, and profits are derived in the appendix.

## 2.4 Equilibrium

The equilibrium in the economy is a set of quantities of labor $\{L(s^t), L^\star(s^t)\}$; consumption $\{C(s^t), C^\star(s^t)\}$; bond holdings $\{B(s^t), B^\star(s^t)\}$; investment $\{X(s^t), X^\star(s^t)\}$; output $\{D(s^t), D^\star(s^t)\}$, capital choices $\{K_0(s^t), K_0^\star(s^t), K_1(s^t), K_1^\star(s^t)\}$, number of exporters and affiliates $\{N^X(s^t), N^{X\star}(s^t), N^F(s^t), N^{F\star}(s^t)\}$, aggregate profits $\{\Pi(s^t), \Pi^\star(s^t)\}$, and prices $q(s^t), P_h(s^t), P_f(s^t), P_h^\star(s^t), P_f^\star(s^t), P(s^t), P^\star(s^t)$; wages $W(s^t), W^\star(s^t)$ such that in each country and in each period

1. Households bond holdings first order condition (foc) and labor supply foc are satisfied
2. Households' budget constraint are satisfied
3. Firms' investment foc holds
4. International bond market is in equilibrium
5. Labor markets are in equilibrium
6. All intermediate goods markets, and the final good markets are in equilibrium

7. Marginal exporters and MP firms' conditions are satisfied

Mathematical derivations for the model equations are in the appendix.

# 3 Calibration

The model is calibrated to mimic key features of extensive and intensive margins of trade and multinational production in the United States and its business cycles at quarterly frequency. A list of these parameters and their values under the benchmark calibration are given in Table 1.

I set fifteen parameters based on their values in existing literature. I set $\beta$ equal to $\frac{1}{1+r/4}$, where annual real return $r = 4\%$. Consumption share in composite commodity, $\gamma$ equals 0.303 and intertemporal elasticity equals two as in Alessandria and Choi [2007]. Capital share and the depreciation rate for capital is standard across growth and business cycle literature: $\alpha = 0.36$ and $\delta_k = 0.025$. Domestic and international elasticity parameters, $\theta$ and $\rho$, are set to 0.8 and 0.33; own and cross aggregate shock persistence parameters are 0.95 and zero respectively; and the standard errors of the innovation to aggregate shocks are set to 0.007, all following Alessandria and Choi [2007]. For the last of these parameters, $\zeta$, there is no precedence in the literature. Cravino and Levchenko estimate it to be between 20-40%, but their model does not map one-to-one with the model in this paper. In the baseline, I assume that $\zeta = 30\%$ and compare the results with technology transfer shut down ($\zeta = 0$).

The remaining six parameters together govern the margins of trade and multinational activity. They are jointly calibrated to match these characteristics of the United States economy. I target the following moments: i. employment share of exporters, ii. employment share of MP firms (parents+affiliates), iii. fraction of firms that export, iv. fraction of firms that conduct MP, v. exporter and MP firms relative entry rates, and vi. exporter productivity premium relative to domestic firms. Most of these moments are well used in the literature. An import share of 15.2% matches United States' import to GDP ratio between 1947 and 2016. Multinational employment share of 26% and fraction of firms being multinationals of 2% is available from Antras and Yeaple [2014]. McCallum and Lincoln [2016] report that 34.4% of US firms were exporters between 1987-2006. The relative entry rate of exporters with respect to MP firms is 18.8% from Deseatnicov and Kucheryavyy [2017]. Finally, the productivity differential between non-domestic and domestic firms is available from Bernard and Jensen [1999], and is between 1.12 and 1.18.

The model does well to match most targeted moments, as seen in Table 2. One particular moment that it misses the target significantly is the exporter productivity premium of 15%. The best combination of parameters in the model generates a 96% productivity premium for exporters. To account for this, the trade-only models are calibrated simi-

Table 1: Benchmark Parameter Values

| Parameter | Description | Value | |
|---|---|---|---|
| | Parameters | | Source |
| $\beta$ | Time preference | 0.99 | Annual return = 4% |
| $\gamma$ | Share of consumption | 0.303 | C/Y in steady state = 78% |
| $\sigma$ | Intertemporal elasticity | 2 | $\in [1, 5]^*$ |
| $\alpha$ | Capital share | 0.36 | $\in [0.35, 0.4]^*$ |
| $\zeta$ | MP technology transfer | 0 | Assumed |
| $\theta$ | Domestic elasticity | 0.8 | $\in [0.7, 0.9]^*$ |
| $\rho$ | International elasticity | 1/3 | Alessandria and Choi [2007] |
| $\delta_k$ | Depreciation rate | 0.025 | $\in [0.02, 0.04]^*$ |
| $M_{11}$, $M_{22}$ | Own persistence | 0.95 | Alessandria and Choi [2007] |
| $M_{12}$, $M_{21}$ | Cross persistence | 0 | Alessandria and Choi [2007] |
| $\Omega_{12}$, $\Omega_{21}$ | SE, aggregate shock | 0.007 | Alessandria and Choi [2007] |
| | Calibrated | | Targets |
| $\delta_h$ | Home and import preference | 0.504 | Import share = 15.2% |
| $\delta_x$ | Import preference | 0.204 | MP empl. sh. = 26% |
| $F^X$ | Export fixed cost | 0.028 | No. exporters = 34.4% |
| $F_0^F$ | MP sunk cost | 0.084 | Frac., MP to X entrants = 18.8% |
| $F_1^F$ | MP fixed cost | 0.056 | No. MP firms = 2% |
| $\sigma_\eta$ | SE, idiosyncratic productivity | 0.627 | Exporter prod. premium = 15% |

*: literature uses values within these bounds, and the parameter is calibrated to the most commonly used value in the range.
Target moments are from Boehm et al. [2019a] and Bernard and Jensen [1999].

larly. As a robustness check, I conduct simulations with alternate trade model calibrations that match fraction of exporting firms, import share of GDP, and exporter productivity premium separately. The results from these simulations are reported in Table 4.

I log-linearize the model around the deterministic steady state, so all the impulse responses are in percentage deviations from this steady state. To get around small sample issues, I simulate the model 1000 times, each for 100 quarters, and the average across 1000 simulations are reported in Table **??**.

**Model tests.** The model matches other key international and domestic correlations observed in the data. All moments except the correlation between real exchange rate and net exports in column 2 of Table **??** qualitatively match the data counterparts in column 1. Importantly, the model generates a real exchange rate depreciation, a worsening trade deficit, and an improvement in the terms of labor in the country hit with a favorable productivity shock. In this it defers from the closest MP model (Contessi [2010]). This difference is a result of including capital, endogenous labor supply, and international risk sharing in the model. Home, for instance, gets a favorable terms of labor when

Table 2: Calibration - Model and the Data

| **Moment** | **Data** | **Model** |
|---|---|---|
| Import share | 15.2% | 15.2% |
| MP employment share | 26% | 26.2% |
| No. exporters | 34.4% | 27.8% |
| Fraction, Number of MP to exporter entry | 18.8% | 13.2% |
| No. MP firms | 2% | 2.8% |
| Exporter productivity premium | [1.12,1.18] | 1.96 |

productivity shock is hit because a part of the associated increase (decrease in Foreign) in wage is absorbed by the positively sloped labor supply curves. In contrast, nominal wages fluctuate much more when labor supply curve is vertical to the extent that Home terms of labor worsens in Contessi [2010]. For this reason, goods are expensive in Home in spite of a favorable productivity shock and the real exchange rate appreciates.

# 4 Results

In this section, I show that the resource transfer channel mitigates, and the technology transfer channel enhances comovement. For exposition, I first describe the effect of MP on comovement and list all the model mechanisms. I describe these mechanisms in greater detail with impulse responses in Figure 1 and Figure 2.

**Aggregate variables.** How do multinational firms affect comovement of output? Column 2-4 of Table **??** reports aggregate variables in the benchmark model, the zero technology transfer case, and the no MP model respectively. From the section on international complements at the bottom of the table, the following results are clear. Taking the no-MP model as a reference, I find that multinational activity leads to greater comovement of output only when one accounts for the within-firm technology transfer. In the no-MP model, output comovement is 0.19 which is lower than than benchmark model value of 0.29. In the alternative model where technology transfer is not accounted for, the comovement measure falls from 0.29 in the benchmark model to 0.08. This implies that multinational firms are an important avenue via which international business cycles are synchronised, and technology transfer within these firms plays an important role.

To further understand the mechanisms, I evaluate the role of entry and exit of exporters and MP firms. In the case of MP firms, shutting down the extensive margin results in lesser transfer of capital across borders. Columns 5 and 6 test how the important variables change when MP and trade extensive margins are shut down respectively. I find that in each case the comovement measure falls by 0.03-0.04 percentage points compared to the benchmark model. At 14%, this is a significant fraction of a total fall in comovement of 0.21 between the benchmark and zero technology transfer models. It is

Table 3: Comparison of Business Cycle Statistics

| | Data | Benchmark | No Tech. Transfer | No MP | No MP Ext. Margin | No Trade Ext. Margin |
|---|---|---|---|---|---|---|
| **Standard deviation (in percent)** | | | | | | |
| Y | 1.72 | 1.40 | 1.51 | 1.30 | 1.29 | 1.27 |
| nx | 0.46 | 0.15 | 0.19 | 0.03 | 0.10 | 0.13 |
| **Standard deviation (relative to output)** | | | | | | |
| C | 0.79 | 0.37 | 0.36 | 0.40 | 0.39 | 0.37 |
| X | 3.25 | 3.67 | 3.76 | 3.34 | 3.47 | 3.63 |
| L | 0.85 | 0.47 | 0.47 | 0.46 | 0.46 | 0.47 |
| q | 2.81 | 0.33 | 0.40 | 0.46 | 0.39 | 0.32 |
| **Domestic correlation with output** | | | | | | |
| C | 0.83 | 0.96 | 0.96 | 0.97 | 0.96 | 0.95 |
| X | 0.93 | 0.98 | 0.97 | 0.99 | 0.98 | 0.98 |
| L | 0.85 | 0.98 | 0.98 | 0.99 | 0.98 | 0.98 |
| nx | -0.38 | -0.50 | -0.58 | -0.48 | -0.48 | -0.49 |
| q | 0.16 | 0.58 | 0.66 | 0.59 | 0.58 | 0.59 |
| q,nx | 0.07 | -0.71 | -0.73 | -0.62 | -0.65 | -0.67 |
| **First Order Autocorrelations** | | | | | | |
| Y | 0.87 | 0.91 | 0.91 | 0.92 | 0.92 | 0.91 |
| C | 0.91 | 0.96 | 0.95 | 0.96 | 0.96 | 0.96 |
| X | 0.84 | 0.87 | 0.86 | 0.89 | 0.88 | 0.87 |
| L | 0.95 | 0.87 | 0.86 | 0.88 | 0.88 | 0.87 |
| nx | 0.90 | 0.88 | 0.88 | 0.95 | 0.92 | 0.90 |
| q | 0.81 | 0.94 | 0.94 | 0.96 | 0.95 | 0.94 |
| **International correlations** | | | | | | |
| Y | 0.51 | 0.29 | 0.08 | 0.19 | 0.17 | 0.26 |
| C | 0.32 | 0.60 | 0.39 | 0.36 | 0.50 | 0.63 |
| X | 0.29 | 0.02 | -0.17 | 0.21 | 0.13 | 0.05 |
| L | 0.43 | 0.20 | 0.01 | 0.25 | 0.26 | 0.19 |

[a] Table provides a comparison of important business cycle statistics under different model parameterization. The first column contains these moments for the United States, taken from Kehoe and Perri.

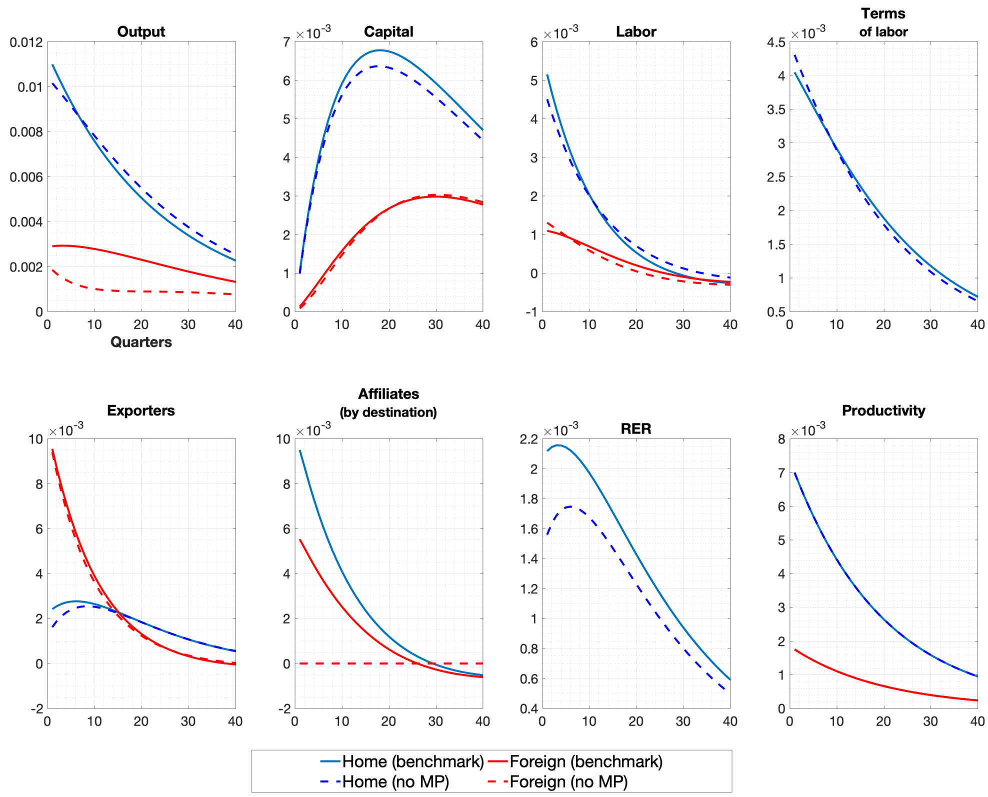


Figure 1: Impulse Responses for the MP and No-MP Models

interesting that MP extensive margins have as large quantitative effect as the extensive margin of exporters in spite of the fact that there are close to 9 times more exporters under these calibrations. This is because of the exporter and MP firms size differential-higher MP sunk costs imply that only the largest firms are multinationals, so them moving production causes large fluctuations in GDP.

In addition to generating higher comovement of output, multinational activity generates greater output volatility. Under the benchmark calibration, the standard deviation of output is 1.4 percent, which is higher than the no-MP standard deviation of 1.3 percent. Volatility is even higher in the MP model (1.51 percent) without technology transfer. The result that multinationals leads to higher volatility is established in the literature. Kalemli-Ozcan et al. [2014], for example, show using a firm level dataset that countries and regions within Europe with greater multinational activity saw greater volatility of output. They argue that investors which are diversified internationally seek greater return from multinationals and are therefore willing to take greater risks. This paper provides an alternative mechanism - multinationals moving capital and technology across borders creates greater volatility.

In what follows, I describe the model mechanisms in detail. Figure 1 compares impulse responses in the benchmark and a reference no-MP economy, and Figure 2 compares benchmark with a zero technology transfer calibration.

**Model mechanisms.** There are three channels in the benchmark model that affect comovement. First, and the source of the resource transfer channel, there is a net inflow of MP firms into the country that experiences an aggregate improvement (Home). As these affiliates bring in capital, there is a net inflow of physical capital to Home. Home output expands because there is greater economic activity with more resources (i.e., capital), and because of the greater number of varieties being produced. Under the benchmark calibration, there is a 0.6% increase in mass of Home owned affiliates in Foreign, and a 1% increase in the mass of Foreign owned affiliates in Home. The no-MP economy, by definition, does not generate any variation in the number of affiliates, so the dashed impulse response lines in subfigure 6 is flat at zero.

The increase in Foreign affiliates in Home is driven by both static and dynamic factors. These firms pay higher fixed cost compared to export fixed cost (and new affiliates pay sunk cost), but they are compensated for by higher operational profits due to more favorable terms of labor in Home. To show this more clearly, I plot Home terms of labor defined as the effective wage rate in Foreign relative to Home:

$$Tol = \frac{q(s^t)W^\star(s^t)}{Z^\star(s^t)} \times \frac{Z(s^t)}{W(s^t)}$$

Subfigure 4 Figure 1 shows Home terms of labor to improve by 0.4% upon impact. This discourages firms from producing abroad to sell in Home. In the net, there is an increase in the mass of firms operating in Home as more Foreign owned affiliates enter compared to the increase in Home owned affiliates. In addition to improving static profits, entering multinationals see a benefit in paying the MP sunk cost when it is cheaper. , the associated fall in MP entry and exit cutoffs in Foreign increase the chances of staying on as a multinational in the future, so the value of conducting MP increases. Given everything else, net inflow of affiliates towards Home results in a fall in output comovement.

Second, like in the trade models, entry and exit of exporters contributes to comovement (solid lines in subfigure 5, Figure 1). To export or not is a static decision. The number of Home exporters increases because cheaper production costs increase their profits abroad. Consequently, high productivity non-exporter and non-MP firms that previously could not cover the export fixed cost can do so when profits are higher. Foreign firms on the other hand benefit from an increased demand from Home which sufficiently counteracts their productivity deficit. In effect, in both countries while the MP entry and exit cutoffs fall, export entry cutoffs fall sufficiently to raise the overall number of exporters. The increase in mass of Home exporters means that the foreign final good producer has more varieties to choose from. As shown in Liao and Santacreu [2015], this "variety-effect" is quantitatively important and contributes to an increase in Foreign output. The no-MP model generates qualitatively the same dynamics of the mass of exporting firms as in the benchmark case. Both Home and Foreign see an increase in exporters.

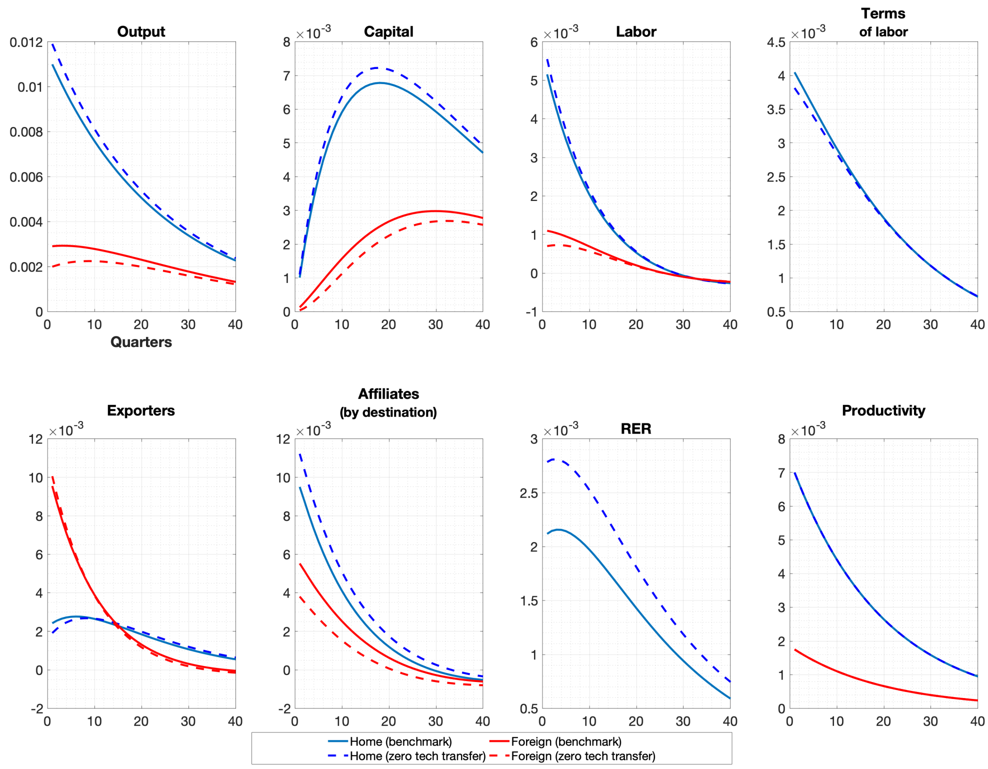


Figure 2: Impulse responses with Different Technology Transfer Levels

Third, complete markets and international risk sharing result in another type of resource transfer channel towards Home driven by a relative rise in investments in Home. The differences in investments is driven by a persistent Home productivity shock that promises higher returns into the future. This leads to Home accumulating a bigger stock of capital than Foreign, so the two countries output comove negatively as a result (Kose and Yi [2006]).

The resource transfer channel via multinationals, a focus of this paper, therefore differs from those in trade models where households own capital stock (Kose and Yi [2006]). The mechanism in trade-only models is gradual and is a result of household investment decisions: Home households borrow internationally to invest more while the Foreign households lower their investments, and the two countries' stock of capital diverge over time. This too reduces output comovement, as pointed out in Kose and Yi [2006]. Under the zero technology transfer calibration in this paper, the traditional resource transfer channel is further strengthened by multinationals as transfer of capital stock towards Home is instantaneous.

**Role of technology transfer**: As described in the note on aggregate variables, within multinational technology transfer is an important mechanism in generating comovement and reducing output volatility. In addition to affecting productivities of existing affiliates, level of technology transfer changes marginal exporter and MP firms entry/exit decisions. This is more evident in subfigure 6 in Figure 2 which plots the path of affiliates under the two technology transfer regimes. When technology transfer is set to zero, Home MP firms are more likely to exit and become exporters. Immediately after the shock hits, Home MP firms increase by only 0.4% in the no technology transfer case compared to benchmark. On the other hand, Foreign MP firms jump by a larger magnitude- 1.2% in the zero technology transfer case compared to 1% in the benchmark case. These differences persist over time, as dashed red (blue) line is above (below) their solid counterparts, which generate an economically significant difference in comovement in the two cases.

The intuition for the differences in firm dynamics is as follows. With higher technology transfer, Foreign affiliates of Home MP firms are compensated for their loss in terms labor with higher aggregate productivity from Home. More Home firms conduct MP as a result. On the contrary, Home affiliates of Foreign firms are forced to carry their lower parent aggregate productivity, which takes away part of the advantage they derive from lower production cost at Home.

## 5 Conclusions

This paper contributes to the literature by developing a dynamic heterogeneous firm model of trade and horizontal multinational production. Compared to the existing research, the model has several distinguishing features. First, multinational production involves technology transfer from parents to affiliates. Second, there is capital which is owned and augmented by firms. Third, labor is elastically supplied and responds to changes in wage rate over the business cycles.

This paper shows that extensive margin of affiliates is quantitatively at least as important as the extensive margin of exporting in determining output comovement across countries. Overall the model generates a net inflow of affiliates towards the expanding country. This accelerates the flow of resources towards this country, augmenting the "resource-transfer" mechanism emphasized in international business cycle models with trade and international risk sharing. Without technology transfer among multinationals, there is a fall in output comovement compared to trade models. This result is overturned when known levels of technology transfer is included in the model. There is less net inflow of affiliates to the country undergoing an expansion, and the resource transfer channel is weakened. This, in combination with technology shock spillover via existing multinationals, leads to an increase in output comovement.

This paper therefore proposes an alternative channel through which multinational production can lead to higher comovement of output across countries. Even though multinational activity (primarily of horizontal type) has seen a dramatic rise since 1990, much more so than trade during the meantime, international business cycle research has largely ignored multinational production as a spillover channel. Much of this literature has focused on trade, but such models have failed to generate the trade-comovement relationship in the data. This paper proposes entry and exit of multinational affiliates, and the associated flow of technology and resources, as a possible way to rationalize the dramatic rise in output comovement observed post 1990.

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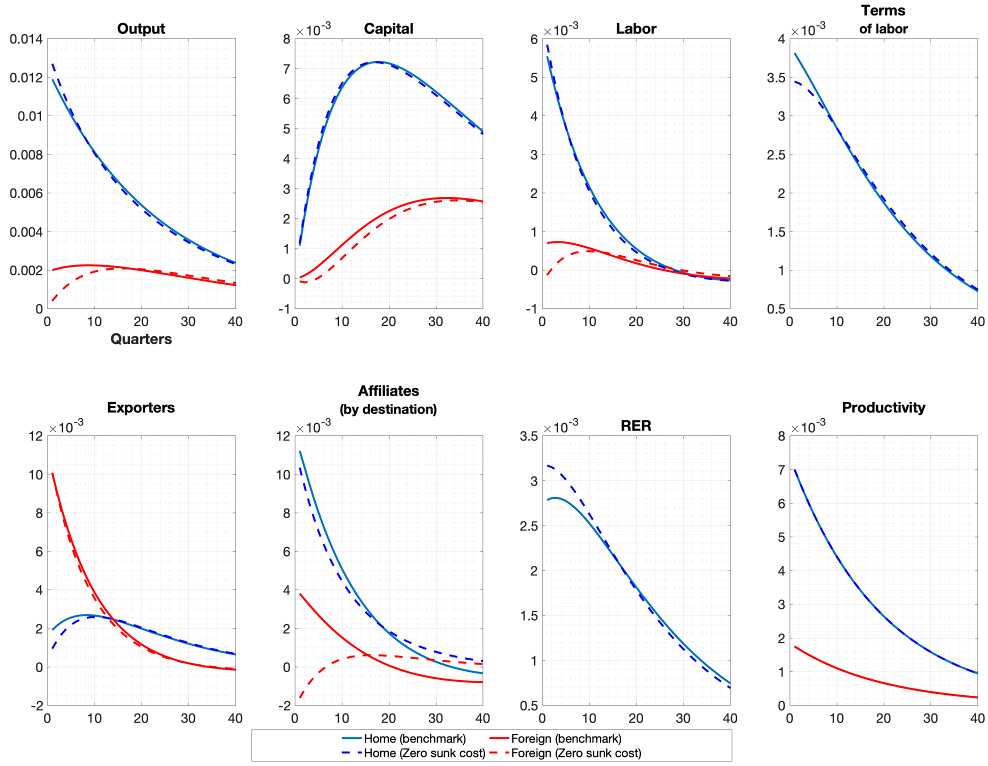


Figure 3: Impulse responses under benchmark calibration and zero MP sunk cost (static MP decisions)

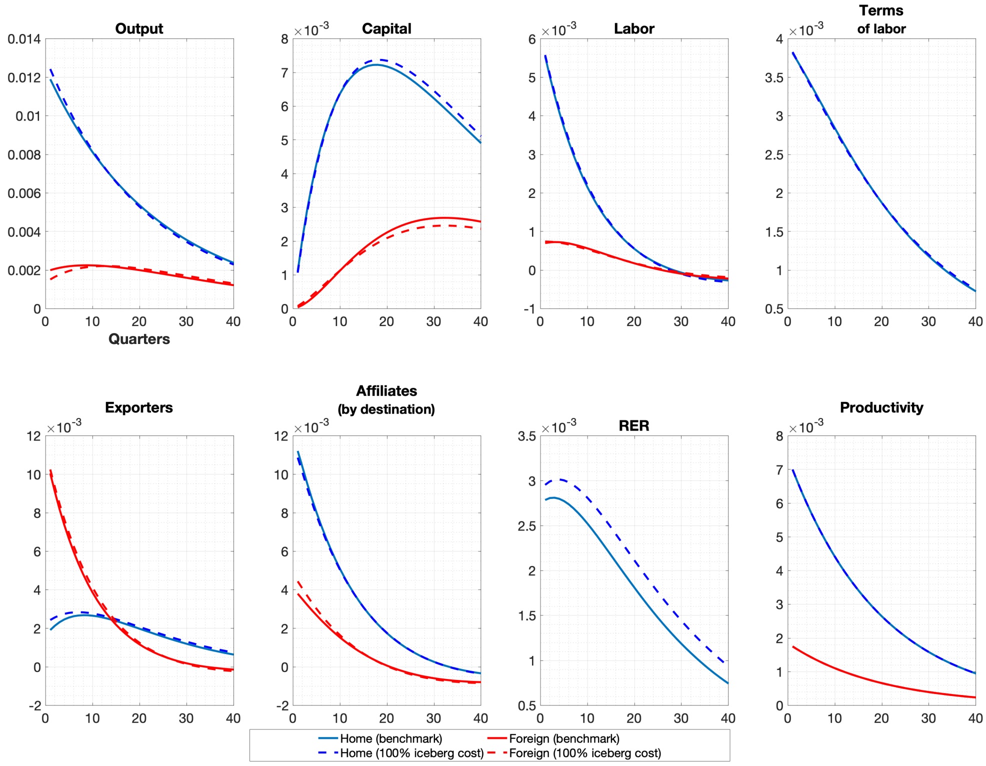


Figure 4: Impulse responses under benchmark calibration and high export iceberg cost

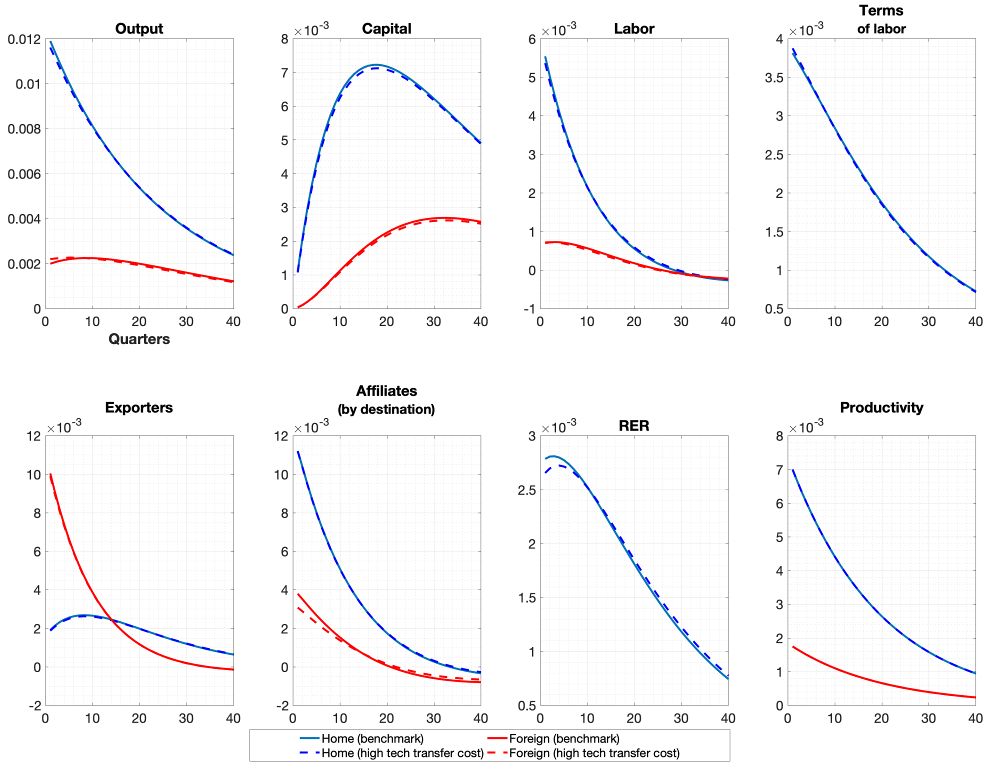


Figure 5: Impulse responses under benchmark calibration and high MP technology transfer cost

# A Technical Appendix

**Firm prices:** Given the monopolistic competitive market structure, an intermediate producer's price in different markets, conditional on the mode of serving those markets, is a constant markup ($= 1/\theta$) above the marginal cost of production:

$$\frac{P_h(i,s^t)}{P(s^t)} = \frac{W(s^t)}{\theta F_l^D(i,s^t)} \tag{17}$$

$$\frac{P^\star_{xf}(i,s^t)}{P^\star(s^t)} = \frac{P_h(i,s^t)}{P(s^t)}\frac{\tau}{q(s^t)} \tag{18}$$

$$\frac{P^\star_{mf}(i,s^t)}{P^\star(s^t)} = \frac{W^\star(s^t)}{\theta F_l^M(i,s^t)} \tag{19}$$

$F_l^D(i,s^t)$, and $F_l^M(i,s^t)$ are marginal labor productivities in the domestic and the foreign plant, $P^\star_{xf}(i,s^t)$ and $P^\star_{mf}(i,s^t)$ are the prices charged by the Home firm $i$ if it exported or if it conducted MP respectively.[9]

**Labor demands:** Because capital is fixed, firms can only adjust the labor in their production function to meet the demand for their variety. For a current non-multinational ($m^{M'}(i,s^t) = 0$), the total labor demand can then be derived by equating the firm output to the demand it faces: $A_{hh}(i,s^t)K(i,s^{t-1})^\alpha L(i,s^t)^{1-\alpha} = y_h(i,s^t) + m^{X'}(i,s^t)\tau y^\star_{xf}(i,s^t)$. Plugging in demands for variety from 3 and 5 and prices 17 and 18 in the rhs, and collecting labor terms together, I get,

$$L_{m^{F'}=0}(i,s^t) = H_{hx}(s^t) \times A(i,s^t)^{\frac{1-\nu}{\alpha}} K(i,s^{t-1})^{1-\nu} \tag{20}$$

where $\nu = (1-\theta)/[1-\theta(1-\alpha)]$ is a constant, and

$$H_{hx}(s^t) = \left[H_h(s^t)^{1/\nu} + m^{X'}(i,s^t)H^\star_x(s^t)^{1/\nu}\right]^\nu \tag{21}$$

$$H_h(s^t) = \left[\delta^{\frac{1}{1-\rho}}\left(\frac{P_h(s^t)}{P(s^t)}\right)^\mu D(s^t)\right]^\nu \left(\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\nu}{\theta-1}} \tag{22}$$

$$H^\star_x(s^t) = \left[\delta_x^{\frac{1}{1-\rho}}\left(\frac{\tau^\theta}{q(s^t)}\right)^{\frac{1}{\theta-1}}\left(\frac{P^\star_{xf}(s^t)}{P^\star(s^t)}\right)^\mu D^\star(s^t)\right]^\nu \left(\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\nu}{\theta-1}} \tag{23}$$

$$H^\star_m(s^t) = \left[(1-\delta_h-\delta_x)^{\frac{1}{1-\rho}}\left(\frac{P^\star_{mf}(s^t)}{P^\star(s^t)}\right)^\mu D^\star(s^t)\right]^\nu \left(\frac{W^\star(s^t)}{\theta(1-\alpha)}\right)^{\frac{\nu}{\theta-1}} \tag{24}$$

are macroeconomic aggregates for Home-owned firms in Home and Foreign respectively. These aggregate do not carry any economic interpretation- they are collections of prices, real wage and the final good output, and come out of the demand functions in 3 and 5 and after collecting the labor terms together.

Next consider a firm that has chosen to be a multinational ($m^{F'}(i,s^t) = 1$). Because it operates in both the countries, I have to specify separately the labor demands in each country. Moreover, the firm can reallocate its capital across the two production locations. Capital allocation affects labor productivities, which impacts the prices charged by the firm and the demand it faces in both the countries. I will take a step back and start by writing labor demands given the capital allocation decision, and then use appropriate first order conditions to find optimal allocations of capital later (see eq. 27 and 29). I will denote by $L^D(i,s^t)$ and $L^{D\star}(i,s^t)$ the labor demands in Home and Foreign for a Home-owned firm. Using the demand for a variety from 3 (given firm's price), and equating it to the production function for MP parents and collecting the labor terms together, I get,

$$L^D_{m^{F'}=1}(i,A^D,K^D) = H_h(s^t)A_{hh}(i,s^t)^{\frac{1-\nu}{\alpha}}K^D(i,s^t)^{1-\nu} \tag{25}$$

[9] It is necessary to distinguish between labor productivities in parent and affiliate entities when conducting MP is a possibility. An MP firm can vary its price by varying these productivities in the two production locations. For example, an MP firm can move capital between the production locations, and thus affect prices in both markets.

Table 4: Alternate No-MP Calibrations and Results

| | Benchmark | No MP- Match Exporter Productivity Differential | No MP- Match Number of Exporters |
|---|---|---|---|
| **Target Moments** | | | |
| MP entry rate | 0.15 | 0.15 | 0.15 |
| MP empl. share | 0.26 | 0.26 | 0.26 |
| Exporter empl. share | 0.34 | 0.34 | 0.28 |
| MP firms | 0.19 | 0.19 | 0.19 |
| Exporters | 0.02 | 0.02 | 0.02 |
| Exporter productivity differential | 1.15 | 1.96 | 1.15 |
| **Main Results** | | | |
| **Standard deviation (in percent)** | | | |
| Y | 1.40 | 67.00 | 1.30 |
| nx | 0.15 | 61.00 | 0.03 |
| **Standard deviation (relative to output)** | | | |
| C | 0.37 | 57.00 | 0.40 |
| X | 3.67 | 65.00 | 3.34 |
| L | 0.47 | 59.00 | 0.46 |
| q | 0.33 | 63.00 | 0.46 |
| **Domestic correlation with output** | | | |
| C | 0.96 | 9.00 | 0.97 |
| X | 0.98 | 24.00 | 0.99 |
| L | 0.98 | 13.00 | 0.99 |
| nx | -0.50 | 15.00 | -0.48 |
| q | 0.58 | 20.00 | 0.59 |
| q,nx | -0.71 | 18.00 | -0.62 |
| **Persistence** | | | |
| Y | 0.91 | 53.00 | 0.92 |
| C | 0.96 | 48.00 | 0.96 |
| X | 0.87 | 52.00 | 0.89 |
| L | 0.87 | 49.00 | 0.88 |
| nx | 0.88 | 50.00 | 0.95 |
| q | 0.94 | 51.00 | 0.96 |
| **International correlations** | | | |
| Y | 0.29 | 26.00 | 0.19 |
| C | 0.60 | 7.00 | 0.36 |
| X | 0.02 | 22.00 | 0.21 |
| L | 0.20 | 11.00 | 0.25 |

[a] Table provides a comparison of important business cycle statistics under different no multinational production parameterizations. Calibrations under columns two through four match exporter productivity differential, number of exporters, and exporter size respectively with the benchmark model.

is the labor demand in the domestic market given the capital $K^D(i,s^t)$ allocated to that market. Similarly, the conditional labor demand for the affiliate can be derived by equating 5 (using MP price 19) to MP affiliate's production function **??**, and collecting the labor terms together,

$$L^{D\star}_{m^{F'}=1}(i, A_{hf}, K^D) = H^\star_m(s^t) A_{hf}(i,s^t)^{\frac{1-\nu}{\alpha}} (K(i,s^{t-1}) - K^D(i,s^t))^{1-\nu} \tag{26}$$

where $A^D(i,s^t)$ and $A^{D\star}(i,s^t)$ are productivities of the MP parent and affiliate defined in 8.

Next, I solve for the capital allocations of an MP firm across parent and affiliate entities. Allocation of capital affects pricing (by changing the marginal product of labor), and hence the demand that the firm faces in each market. Because capital can be transferred freely across borders, it is allocated to maximize joint profits in each period (i.e., a static decision). Condensing the fixed and sunk costs into a single term, the total static profit of a multinational can be written as,

$$\Pi(total) = \Pi(Parent) + \Pi(Affiliate)$$

$$\begin{aligned} &= P_h(i,s^t)y^D(i,s^t) - P(s^t)W(s^t)L^D(i,s^t) \\ &\qquad + e(s^t)\left[P^{F\star}_h(i,s^t)y^F(i,s^t) - P^\star(s^t)W^\star(s^t)L^{D\star}(i,s^t)\right] - FC \end{aligned}$$

Plug the production functions in 2.3.1, and the capital quantity constraint 7 to get,

$$\begin{aligned} \Pi(total) &= P_h(i,s^t)A^D(i,s^t)K^D(i,s^t)^\alpha L^D(i,s^t)^{1-\alpha} - P(s^t)W(s^t)L^D(i,s^t) + \\ &e(s^t)\Big[P^{F\star}_h(i,s^t)A^{D\star}(i,s^t)\left(\bar{K}(i,s^t) - K^D(i,s^t)\right)^\alpha L^{D\star}(i,s^t)^{1-\alpha} - P^\star(s^t)W^\star(s^t)L^{D\star}(i,s^t)\Big] - FC \end{aligned}$$

$$= \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)} P(s^t)W(s^t)\left[L^D(i,s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)} L^{D\star}(i,s^t)\right] - FC$$

Plugging in the labor demand functions in 25 and 26, and differentiating the above function with respect to domestic capital utilization $K^D(i,s^t)$, it can be written as a function of the capital stock that the firm is born with and macroeconomic aggregates in both the markets,

$$K^D_{m^{F'}=1}(i,s^t) = \frac{K(i,s^{t-1})}{1+G(s^t)} \tag{27}$$

where

$$G(s^t) = \left(\frac{q(s^t)W^\star(s^t)}{W(s^t)} \frac{H^\star_m(s^t)}{H_h(s^t)}\right)^{1/\nu} h^{(\nu-1)/\nu\alpha} \exp\left(\frac{(1-\nu)(1-\zeta)(Z^\star(s^t)-Z(s^t))}{\nu\alpha}\right) \tag{28}$$

is another macroeconomic aggregate which corresponds to foreign country's relative cost and demand advantage over the home country for MP firms.

The capital stock allocated to the MP firm's affiliate is given by $K^{D\star}_{m^{F'}=1}(i,s^t) = K(i,s^{t-1}) - K^D_{m^{F'}=1}(i,s^t)$, which equals,

$$K^{D\star}_{m^{F'}=1}(i,s^t) = \frac{G(s^t)}{1+G(s^t)} K(i,s^t) \tag{29}$$

Clearly, the capital utilization functions 27 and 29 are independent of firms' idiosyncratic productivity. This result simplifies computation greatly, because it implies that every MP firm from Home employs the same fraction of its initial stock in the Home market. When the two countries are identical- i.e., they have the same aggregate states, wages, masses of exporters and MP firms, the MP firms use half of their capital in either location. Whenever one of the countries has lower effective wage, that country receives a greater share of the capital.

Note that the labor demands for MP firms derived in 25 and 26 took the capital allocations as given. So we can plug 27 and 29 back into these functions to get labor demands.

The assumption that firm productivity $\eta$ is distributed i.i.d greatly simplifies computation. It implies that every firm has the same expectation for $\eta$ next period. All current MP firms then have the same (and higher) expected value tomorrow on account of having already paid the sunk cost. As a result, these firms choose the same capital stock for tomorrow, denoted $K_1(s^t)$. For the current non-MP firms, the expected value tomorrow is lower, and they invest to have a lower capital stock tomorrow, denoted $K_0(s^t)$.

$$K(i,s^t)=\begin{cases}K_0(s^t) & \text{if } \ m^{F'}(i,s^t)=0\\ K_1(s^t) & \text{if } \ m^{F'}(i,s^t)=1\end{cases} \tag{30}$$

These are the capital stocks that firms are born with in the next period.

**Firm Value:** I will use a ordered triple $(m^F,\, m^{X'},\, m^{F'})$ to summarize the labor and investment choices of a firm depending on their last-period MP status, and current export and MP choices respectively. For example, $L_{0,1,0}(i,s^t)$ corresponds to the labor employed by a last-period non-MP firm, which exports today; $L_{0,0,1}(i,s^t)$ corresponds to labor employed by a last-period non-MP firm, which conducts MP today. The investment variable is defined similarly. Using the order triplet helps to define the firms' value function by activity and last-period MP status 31-33, and to define the export and MP productivity cutoff conditions 34-36.

Using this notation, I can write firms' current value conditional on last-period MP status $m^F$, current productivity $\eta$, and aggregate conditions $s^t$. The value of a domestic-only producer is,

$$\begin{aligned}V^D(\eta,K_{m^F},m^F,s^t)=\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)L_{m^F,0,0}(i,s^t)-P(s^t)x_{m^F,0,0}(i,s^t)\\ +\sum_{s^{t+1}}\sum_{\eta'}Q(s^{t+1}|s^t)Pr(\eta')V(\eta',K_0',0,s^{t+1})\end{aligned} \tag{31}$$

Where the $L_{m^F,0,0}$ can be derived from plugging $m^{X'}=0$ in 20, and $x_{m^F,0,0}(i,s^t)=K_0(s^t)-(1-\delta_k)K_{m^F}(s^{t-1})$ is the investment given capital mass-points 30. The value of an exporter is,

$$\begin{aligned}V^X(\eta,K_{m^F},m^F,s^t)=\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)L_{m^F,1,0}(i,s^t)-\frac{q(s^t)P(s^t)W^\star(s^t)}{\exp\left(Z^\star(s^t)\right)}F^X\\ -P(s^t)x_{m^F,1,0}(i,s^t)+\sum_{s^{t+1}}\sum_{\eta'}Q(s^{t+1}|s^t)Pr(\eta')V(\eta',K_0',0,s^{t+1})\end{aligned} \tag{32}$$

Where the $L_{m^F,1,0}$ can be derived from plugging $m^{X'}=1$ in 20, and $x_{m^F,1,0}(i,s^t)=K_0(s^t)-(1-\delta_k)K_{m^F}(s^{t-1})$ is the investment given capital mass-points 30. And finally, the value of an MP firm today is,

$$\begin{aligned}V^F(\eta,K_{m^F},m^F,s^t)=\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)\left[L^D_{m^F,0,1}(i,s^t)+\frac{q(s^t)W^\star(s^t)}{W(s^t)}L^F_{m^F,0,1}(i,s^t)\right]\\ -\frac{q(s^t)P(s^t)W^\star(s^t)}{\exp\left(\zeta Z(s^t)+(1-\zeta)Z^\star(s^t)\right)}\left[F_1^F+(1-m^F)F_0^F\right]\\ -P(s^t)x_{m^F,0,1}(i,s^t)+\sum_{s^{t+1}}\sum_{\eta'}Q(s^{t+1}|s^t)Pr(\eta')V(\eta',K_1',1,s^{t+1})\end{aligned} \tag{33}$$

Where labor demands $L^D_{m^F,0,1}$ (MP parent's labor demand), and $L^{D\star}_{m^F,0,1}$ (MP affiliate's labor demand) can be inferred from 25 and 26 respectively; $x_{m^F,0,1}(i,s^t)=K_1(s^t)-(1-\delta_k)K_{m^F}(s^{t-1})$ is the firms' investment.

The condition for marginal exporter 15 can be re-written using the domestic and exporters' value functions (31 and 32) as

$$\begin{aligned}0=\frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)\left[L_{m^F,1,0}(i,s^t)-L_{m^F,0,0}(i,s^t)\right]-\frac{q(s^t)P(s^t)W^\star(s^t)}{\exp\left(Z^\star(s^t)\right)}F^X\\ +P(s^t)\left[x_{m^F,0,0}(i,s^t)-x_{m^F,1,0}(i,s^t)\right]\\ +\sum_{s^{t+1}}\sum_{\eta'}Q(s^{t+1}|s^t)Pr(\eta')\left[V(\eta',K_0',0,s^{t+1})-V(\eta',K_0',0,s^{t+1})\right]\end{aligned}$$

Note that capital stock in a period is independent on current export status. As a result, investment is depends only on current last period and current MP choices ($m^F$ and $m^{F'}$). Therefore, $x_{m^F,0,0}(i,s^t)=$

$x_{m^F,1,0}(i, s^t)$.

$$\frac{q(s^t)W^\star(s^t)}{W(s^t)\exp(Z^\star(s^t))}F^X = \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}\left[L_{m^F,1,0}(i,s^t) - L_{m^F,0,0}(i,s^t)\right]$$
$$= \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}A(i,s^t)^{\frac{1-\nu}{\alpha}}K_{m^F}(s^{t-1})^{1-\nu}\left[H^\star_{hx}(s^t) - H_h(s^t)\right] \tag{34}$$

Equation 34 shows that given a firm does not choose to conduct MP, the exporting decision is static.

The marginal continuing MP firm's condition is $0 = V^F(\eta^F_1, K_1, 1, s^t) - V^X(\eta^F_1, K_1, 1, s^t)$. Also, $x_{1,1,0}(i, s^t) = K_0(s^t) - (1-\delta_k)K_1(s^{t-1})$ and $x_{1,0,1}(i, s^t) = K_1(s^t) - (1-\delta_k)K_1(s^{t-1})$. For such a firm,

$$0 = \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)\left[L^D_{m^{F'}=1}(i,s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)}L^{D\star}_{m^{F'}=1}(i,s^t) - L_{1,1,0}(i,s^t)\right]$$
$$- \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp(Z^\star(s^t))}\left[\frac{F^F_1}{\exp(\zeta(Z(s^t) - Z^\star(s^t)))} - F^X\right] - P(s^t)\left[K_1(s^t) - K_0(s^t)\right]$$
$$+ \sum_{s^{t+1}}\sum_{\eta'} Q(s^{t+1}|s^t)Pr(\eta')\left[V(\eta', K'_1, 1, s^{t+1}) - V(\eta', K'_0, 0, s^{t+1})\right] \tag{35}$$

For the marginal new MP firm, $0 = V^F(\eta^F_0, K_0, 0, s^t) - V^X(\eta^F_0, K_0, 0, s^t)$. Which simplifies to,

$$0 = \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)}P(s^t)W(s^t)\left[L^D_{m^{F'}=1}(i,s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)}L^{D\star}_{m^{F'}=1}(i,s^t) - L_{0,1,0}(i,s^t)\right]$$
$$- \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp(Z^\star(s^t))}\left[\frac{F^F_1 + F^F_0}{\exp(\zeta(Z(s^t) - Z^\star(s^t)))} - F^X\right] - P(s^t)\left[K_1(s^t) - K_0(s^t)\right]$$
$$+ \sum_{s^{t+1}}\sum_{\eta'} Q(s^{t+1}|s^t)Pr(\eta')\left[V(\eta', K'_1, 1, s^{t+1}) - V(\eta', K'_0, 0, s^{t+1})\right] \tag{36}$$

**Aggregation**: Given the cutoffs, I can write the laws of motion for number of exporters and MP firms originating in each country, $N^X(s^t)$ and $N^F(s^t)$. Among the non-MP firms last-period, a fraction $1 - \Phi(\eta^F_0(s^t))$ conduct MP today; among MP firms a fraction $1 - \Phi(\eta^F_1(s^t))$ conduct MP today. Then,

$$N^F(s^t) = \left[1 - \Phi(\eta^F_1(s^t))\right]N^F(s^{t-1}) + \left[1 - \Phi(\eta^F_0(s^t))\right](1 - N^F(s^{t-1})) \tag{37}$$

Similarly, among the non-MP firms last-period, a fraction $\Phi(\eta^F_0(s^t)) - \Phi(\eta^X_0(s^t))$ conduct MP today; among MP firms a fraction $\Phi(\eta^F_1(s^t)) - \Phi(\eta^X_1(s^t))$ conduct MP today. Then,

$$N^X(s^t) = \left[\Phi(\eta^F_1(s^t)) - \Phi(\eta^X_1(s^t))\right]N^F(s^{t-1}) + \left[\Phi(\eta^F_0(s^t)) - \Phi(\eta^X_0(s^t))\right]\left[1 - N^F(s^{t-1})\right] \tag{38}$$

Using the numbers of exporters and MP firms, I can write the aggregates for capital, investment, labor demand, and the value of firms.

*Capital:* Among this period firms, all non-MP firms choose a capital stock $K_0$, while all MP firms choose a capital stock $K_1$. Therefore,

$$K(s^t) = \left[1 - N(s^t)\right]K_0(s^t) + N(s^t)K_1(s^t) \tag{39}$$

*Investment:* Aggregate investment at Home is then a difference between the total capital stock chosen today, and the undepreciated capital from last period:

$$X(s^t) = K(s^t) - (1-\delta_k)K(s^{t-1}) \tag{40}$$

*Labor demands in home*: Home labor is utilized for: 1. production by Home-owned firms for domestic production and exporting, and by Home affiliates of Foreign firms, and 2. fixed cost payments by Home exporters and by Home affiliates of Foreign firms. I will compute the average labor demand in Home and Foreign for production by last-period Home-owned non-MP and MP firms, $L^D_0(s^t)$, $L^D_1(s^t)$, $L^F_0(s^t)$, and $L_1^F(s^t)$:

$$L_0^D(s^t) = \frac{\int_{i \notin \xi^F(s^{t-1})} L(i,s^t)di}{1 - N^F(s^{t-1})}, \qquad L_1^D(s^t) = \frac{\int_{i \in \xi^F(s^{t-1})} L(i,s^t)di}{N^F(s^{t-1})}$$

$$L_0^{D\star}(s^t) = \frac{\int_{i \notin \xi^F(s^{t-1})} L^{D\star}_{m^{F'}=1}(i,s^t)di}{1 - N^F(s^{t-1})}, \qquad L_1^{D\star}(s^t) = \frac{\int_{i \in \xi^F(s^{t-1})} L^{D\star}_{m^{F'}=1}(i,s^t)di}{N^F(s^{t-1})}$$

Because $\eta$ is distributed i.i.d, last period MP- and non-MP firms have the same distribution today. In other words, the probability that a firm with a productivity $\eta$ today was an MP-firm last period is $N^F(s^{t-1})$. Conditional on last-period MP status $m^F$, I can write the average labor in Home by Home firms as as a sum of domestic-only firms' labor demand (integrate 20 with $m^{X'} = 0$ over $-\infty$ to $\eta^X_{m^F}$), exporters' labor demand (integrate 20 with $m^{X'} = 1$ over $\eta^X_{m^F}$ to $\eta^F_{m^F}$), and the Home labor demand by Home owned MP firms (integrate 25 over $\eta^F_{m^F}$ to $\infty$), divided by the number of firms with MP status $m^F$ last-period. This simplifies to,

$$L^D_{m^F}(s^t) = K_{m^F}(s^{t-1})^{1-\nu} \exp\left(Z(s^t)(1-\nu)/\alpha\right) \times \Bigg\{ H_h(s^t) \int_{-\infty}^{\eta^X_{m^F}} \exp\left(\frac{\eta(1-\nu)}{\alpha}\right)\Upsilon(\eta)d\eta$$
$$+ H^\star_{hx}(s^t) \int_{\eta^X_{m^F}}^{\eta^F_{m^F}} \exp\left(\frac{\eta(1-\nu)}{\alpha}\right)\Upsilon(\eta)d\eta + \frac{H_h(s^t)}{[1+G(s^t)]^{1-\nu}} \int_{\eta^F_{m^F}}^{\infty} \exp\left(\frac{\eta(1-\nu)}{\alpha}\right)\Upsilon(\eta)d\eta \Bigg\} \quad (41)$$

Where $\Upsilon(\eta)$ is the p.d.f of $\eta$. The average of Home-owned MP firms' labor demand in Foreign, conditional on last-period MP status $m^F$, is the sum of 26 between $\eta^F_{m^F}$ to $\infty$, divided by the number of firms with MP status $m^F$ last-period,

$$L^{D\star}_{m^F}(s^t) = \left[K_{m^F}(s^{t-1}) \frac{G(s^t)}{1+G(s^t)}\right]^{1-\nu} \left(\frac{\exp\left(\zeta Z(s^t) + (1-\zeta)Z^\star(s^t)\right)}{h}\right)^{\frac{1-\nu}{\alpha}}$$
$$\times H^\star_m(s^t) \int_{\eta^F_{m^F}}^{\infty} \exp\left(\frac{\eta(1-\nu)}{\alpha}\right)\Upsilon(\eta)d\eta \quad (42)$$

The total demand in Home labor is then a sum of the operational labor demands and fixed cost labor demands by firms producing in Home:

$$L(s^t) = \left[1 - N^F(s^{t-1})\right] L_0^D(s^t) + N^F(s^{t-1}) L_1^D(s^t) + \left[1 - N^{F\star}(s^{t-1})\right] L_0^F(s^t)$$
$$+ N^{F\star}(s^{t-1}) L_1^F(s^t) + L_{fc}(s^t) \quad (43)$$

where $L_{fc}(s^t)$ is the total fixed costs paid in Home labor, which includes fixed cost payments by Foreign exporters, and fixed and sunk cost payments by Foreign MP firms.

$$L_{fc}(s^t) = \frac{F^X}{\exp\left(Z(s^t)\right)} N^{X\star}(s^t) + \int_{i \in \xi^{F\star}(s^t)} \left\{ \frac{[1 - m^{F\star}(i,s^{t-1})](F_1^F + F_0^F) + m^{F\star}(i,s^{t-1}) F_1^F}{\exp\left(\zeta Z^\star(s^t) + (1-\zeta)Z(s^t)\right)} \right\} di$$

Because firm productivity is distributed i.i.d., firms' productivity today is independent of their productivity last period. As a result, a constant fraction $N^F(s^{t-1})$ of today's MP firms conducted MP last period. These MP firms do not pay the sunk cost today, while the other MP firms do,

$$L_{fc}(s^t) = \frac{F^X}{\exp\left(Z(s^t)\right)} N^{X\star}(s^t) + \frac{F_1^F + F_0^F}{\exp\left(\zeta Z^\star(s^t) + (1-\zeta)Z(s^t)\right)} \left(1 - N^{F\star}(s^{t-1})\right) \left[1 - \Phi(\eta_0^{F\star}(s^t))\right]$$
$$+ \frac{F_1^F}{\exp\left(\zeta Z^\star(s^t) + (1-\zeta)Z(s^t)\right)} N^{F\star}(s^{t-1}) \left[1 - \Phi(\eta_1^{F\star}(s^t))\right] \quad (44)$$

*Profits:* Let $\Pi_{m^F}(s^t)$ be the aggregate profit of all firms with MP status $m^F$ last period.

$$\Pi_0(s^t) = \frac{\int_{i \notin \xi^F(s^{t-1})} \Pi(i,s^t)di}{1 - N^F(s^{t-1})}, \qquad \Pi_1(s^t) = \frac{\int_{i \in \xi^F(s^{t-1})} \Pi(i,s^t)di}{N^F(s^{t-1})}$$

Given $m^F$, $\Pi_{m^F}(s^t)$ is the sum of domestic firms' profits 12 integrated between $-\infty$ to $\eta^X_{m^F}$, exporters' profits 13 integrated between $\eta^X_{m^F}$ to $\eta^F_{m^F}$, and MP firms' profits 14 integrated between $\eta^F_{m^F}$ to $\infty$, divided by the number of firms with MP status $m^F$ last-period,

$$\begin{aligned}\Pi_{m^F}(s^t) = \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)} P(s^t)W(s^t)\left[L^D_{m^F}(s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)} L^{D\star}_{m^F}(s^t)\right] \\ - \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp\left(Z^\star(s^t)\right)}\left\{F^X\left[\Phi(\eta^F_{m^F}) - \Phi(\eta^X_{m^F})\right] + \frac{F^F_1 + (1-m^F)F^F_0}{\exp\left(\zeta(Z(s^t)-Z^\star(s^t))\right)}\left[1-\Phi(\eta^F_{m^F})\right]\right\} \\ - P(s^t)\left\{[1-\Phi(\eta^F_{m^F})]K_1(s^t) + \Phi(\eta^F_{m^F})K_0(s^t) - (1-\delta_k)K_{m^F}(s^{t-1})\right\}\end{aligned}$$

The total profits of all firms originating in Home, including the profits of affiliates earned abroad, is,

$$\Pi(s^t) = [1-N^F(s^{t-1})]\Pi_0(s^t) + N^F(s^{t-1})\Pi_1(s^t) \tag{45}$$

*Prices:* Before I aggregate firm prices, I will rewrite firm prices in 17-19. Plugging into 17 the marginal product of labor $F^D_l(i,s^t) = (1-\alpha)A_{hh}(i,s^t)\left(\frac{K(i,s^{t-1})}{L(i,s^t)}\right)^\alpha$, where $L(i,s^t)$ is given by 20, I get the Home price of domestic firms and exporters,

$$\left(\frac{P_{m^{F\prime}=0}(i,s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} = \left(\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} A_{hh}(i,s^t)^{\frac{1-\nu}{\alpha}} K_{m^F}(s^{t-1})^{1-\nu} H^{\frac{\nu-1}{\nu}} \tag{46}$$

Note that the price is raised to the power $\theta/(\theta-1)$, which results in the exponents on the right hand side.

MP firms, on the other hand, use only a fraction of their capital in the parent firm. This affects the marginal product of labor, giving the following expression for their Home price,

$$\left(\frac{P^D_{m^{F\prime}=1}(i,s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} = \left(\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} A_h(i,s^t)^{\frac{1-\nu}{\alpha}} \left(\frac{K_{m^F}(s^{t-1})}{1+G(s^t)}\right)^{1-\nu} H_h(s^t)^{\frac{\nu-1}{\nu}} \tag{47}$$

The CES aggregate of the Home prices of Home firms (raised to exponent $\theta/(\theta-1)$) is then,

$$\left(\frac{P_h(s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} = \int_i \left(\frac{P_h(i,s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} di$$

Which equals the sum of the firm prices integrated over the productivity ranges for domestic firms (with $m^{X\prime} = 0$ in 46) and exporters (with $m^{X\prime} = 1$ in 46), and 47 summed over the productivity range for MP firms,

$$\left(\frac{P_h(s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} = \left[\int_{i\in\xi^D(s^t)} + \int_{i\in\xi^X(s^t)} + \int_{i\in\xi^F(s^t)}\right]\left(\frac{P_h(i,s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} di$$

Because the productivity ranges and the capital stocks depend on last-period MP status, I integrate above expression separately for last-period MP firms ($N^F(s^{t-1})$ in number) and non-MP firms ($1-N^F(s^{t-1})$ in number), and add them together,

$$
\begin{aligned}
\left(\frac{P_h(s^t)}{P(s^t)}\right)^{\frac{\theta}{\theta-1}} &= \left(\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} \exp\left(Z(s^t)(1-\nu)/\alpha\right) \times \Bigg\{K_0(s^{t-1})^{1-\nu}[1-N^F(s^{t-1})] \\
&\Bigg[H_h^{\frac{\nu-1}{\nu}} \int_{-\infty}^{\eta_0^X(s^t)} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta) + H_{hx}^{\star\frac{\nu-1}{\nu}} \int_{\eta_0^X(s^t)}^{\eta_0^F(s^t)} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta) + \\
&\frac{H_h^{\frac{\nu-1}{\nu}}}{(1+G(s^t))^{1-\nu}} \int_{\eta_0^F(s^t)}^{\infty} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta)\Bigg] \\
&+ K_1(s^{t-1})^{1-\nu} N^F(s^{t-1}) \Bigg[H_h^{\frac{\nu-1}{\nu}} \int_{-\infty}^{\eta_1^X(s^t)} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta) + H_{hx}^{\star\frac{\nu-1}{\nu}} \int_{\eta_1^X(s^t)}^{\eta_1^F(s^t)} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta) \\
&+ \frac{H_h^{\frac{\nu-1}{\nu}}}{(1+G(s^t))^{1-\nu}} \int_{\eta_1^F(s^t)}^{\infty} \exp\left(\frac{1-\nu}{\alpha}\eta\right)\Upsilon(\eta)\Bigg]\Bigg\} \qquad (48)
\end{aligned}
$$

To derive the the Home exporters and MP firms' price index in Foreign, I will rewrite their individual firm prices. Exporters charge a markup over their domestic price, adjusted for the exchange rate,

$$
\frac{P_{xf}^\star(i,s^t)}{P^\star(s^t)} = \frac{P_h(i,s^t)}{P(s^t)} \frac{\tau}{q(s^t)} \qquad (49)
$$

The aggregate exporter price index is then,

$$
\begin{aligned}
\left(\frac{P_{xf}^\star(s^t)}{P^\star(s^t)}\right)^{\frac{\theta}{\theta-1}} &= \int_{i\in\xi^X(s^t)} \left(\frac{P_{xf}^\star(i,s^t)}{P^\star(s^t)}\right)^{\frac{\theta}{\theta-1}} di \\
&= \left(\frac{\tau}{q(s^t)}\frac{W(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} H_{hx}^{\star\frac{\nu-1}{\nu}} \exp\left(\frac{Z(s^t)(1-\nu)}{\alpha}\right)\Bigg\{K_0(s^{t-1})^{1-\nu}[1-N^F(s^{t-1})] \\
&\times \int_{\eta_0^X}^{\eta_0^F} \exp\left(\eta(1-\nu)/\alpha\right)\Upsilon(\eta)d\eta + K_1(s^{t-1})^{1-\nu}N^F(s^{t-1}) \int_{\eta_1^X}^{\eta_1^F} \exp\left(\eta(1-\nu)/\alpha\right)\Upsilon(\eta)d\eta\Bigg\} \qquad (50)
\end{aligned}
$$

And the price charged in the foreign market by an MP firm is obtained by plugging in $F_l(i,s^t) = (1-\alpha)A_{hf}(i,s^t)\left(\frac{L^{D\star}(i,s^t)}{K^{D\star}(i,s^t)}\right)^{1-\alpha}$ into 19, where $L^{D\star}(i,s^t)$ and $K^{D\star}(i,s^t)$ are given by 26 and 29,

$$
\left(\frac{P_{mF'=1}^{D\star}(i,s^t)}{P^\star(s^t)}\right)^{\frac{\theta}{\theta-1}} = \left(\frac{W^\star(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} A_{hf}(i,s^t)^{\frac{1-\nu}{\alpha}} K_{mF'=1}^{D\star}(i,s^t)^{1-\nu} H_h^\star(s^t)^{\frac{\nu-1}{\nu}} \qquad (51)
$$

The aggregate of multinational affiliates' price in Foreign is then,

$$
\begin{aligned}
\left(\frac{P_{mf}^\star(s^t)}{P^\star(s^t)}\right)^{\frac{\theta}{\theta-1}} &= \int_{i\in\xi^M(s^t)} \left(\frac{P_{mf}^\star(i,s^t)}{P^\star(s^t)}\right)^{\frac{\theta}{\theta-1}} di \\
&= \left(\frac{W^\star(s^t)}{\theta(1-\alpha)}\right)^{\frac{\theta}{\theta-1}} \left(\frac{\exp\left(\zeta Z(s^t)+(1-\zeta)Z^\star(s^t)\right)}{h}\right)^{\frac{1-\nu}{\alpha}} H_m^\star(s^t)^{\frac{\nu-1}{\nu}} \left(\frac{G(s^t)}{1+G(s^t)}\right)^{1-\nu} \\
&\times \Bigg\{K_0(s^{t-1})^{1-\nu}[1-N^F(s^{t-1})] \int_{\eta_0^F}^{\infty} \exp\left(\eta(1-\nu)/\alpha\right)\Upsilon(\eta)d\eta \\
&+ K_1(s^{t-1})^{1-\nu}N^F(s^{t-1}) \times \int_{\eta_1^F}^{\infty} \exp\left(\eta(1-\nu)/\alpha\right)\Upsilon(\eta)d\eta\Bigg\} \qquad (52)
\end{aligned}
$$

aggregate price 6 can then be rewritten as,

$$1 = \delta_h^{\frac{1}{1-\rho}} \left(\frac{P_h(s^t)}{P(s^t)}\right)^{\frac{\rho}{\rho-1}} + \delta_x^{\frac{1}{1-\rho}} \left(\frac{P_{xf}(s^t)}{P(s^t)}\right)^{\frac{\rho}{\rho-1}} + (1-\delta_h-\delta_x)^{\frac{1}{1-\rho}} \left(\frac{P_{mf}(s^t)}{P(s^t)}\right)^{\frac{\rho}{\rho-1}} \tag{53}$$

where the individual price ratios within the above equation are given by 48 and the foreign equivalents of 50 and 52 (which is the price relevant for the Home final good aggregator).

*Firm Value and Cutoffs:* I specify the cutoff conditions 34, 35, and 36 in terms of average values that can be tracked over time. I define $V_{m^F}(s^t)$ as the average value today of all firms that had the MP status $m^F$ last period.

$$V_0(s^t) = \frac{1}{1-N^F(s^{t-1})} \int_{i \notin \xi^F(s^{t-1})} V(\eta, K_0(s^{t-1}), 0, s^t) di$$
$$V_1(s^t) = \frac{1}{N^F(s^{t-1})} \int_{i \in \xi^F(s^{t-1})} V(\eta, K_1(s^{t-1}), 1, s^t) di$$

$V_{m^F}(s^t)$ is then a sum of $V^D(i, s^t)$ in 31 over the productivity range of domestic firms, $V^X(i, s^t)$ in 32 over the productivity range of exporters, and $V^F(i, s^t)$ in 33 over the productivity range of MP firms, each with last-period MP status $m^F$. In general form, this simplifies to,

$$\begin{aligned}
V_{m^F}(s^t) = & \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)} P(s^t) W(s^t) \left[L^D_{m^F}(s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)} L^{D\star}_{m^F}(s^t)\right] \\
& - \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp(Z^\star(s^t))} \left[\{\Phi(\eta^F_{m^F}) - \Phi(\eta^X_{m^F})\} F^X + \{1-\Phi(\eta^F_{m^F})\} \frac{F^F_1 + (1-m^F)F^F_0}{\exp(\zeta(Z(s^t) - Z^\star(s^t)))}\right] \\
& - P(s^t)\left\{\Phi(\eta^F_{m^F}) K_0(s^t) + [1-\Phi(\eta^F_{m^F})] K_1(s^t)\right\} + P(s^t)(1-\delta_k) K_{m^F}(s^{t-1}) \\
& + \sum_{s^{t+1}} Q(s^{t+1}|s^t) \left\{\Phi(\eta^F_{m^F}) V_0(s^{t+1}) + [1-\Phi(\eta^F_{m^F})] V_1(s^{t+1})\right\}
\end{aligned} \tag{54}$$

I will now compute the difference in average values today of last period MP and non-MP firms. This difference can be plugged into modified MP cutoff conditions 36 and 35 to derive cutoff conditions that are tractable over time,

$$\begin{aligned}
V_1(s^t) - V_0(s^t) = & \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)} P(s^t) W(s^t) \\
& \times \left[L^D_1(s^t) - L^D_0(s^t) + \frac{q(s^t)W^\star(s^t)}{W(s^t)} \left(L^{D\star}_1(s^t) - L^{D\star}_0(s^t)\right)\right] \\
& - \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp(Z^\star(s^t))} \Big[F^X \left\{\Phi(\eta^F_1) + \Phi(\eta^X_0) - \Phi(\eta^F_0) - \Phi(\eta^X_1)\right\} \\
& + \frac{\{F^F_1[1-\Phi(\eta^F_1)] - [F^F_1 + F^F_0][1-\Phi(\eta^F_0)]\}}{\exp(\zeta(Z(s^t) - Z^\star(s^t)))}\Big] \\
& + [\Phi(\eta^F_0) - \Phi(\eta^F_1)] \sum_{s^{t+1}} Q(s^{t+1}|s^t)[V_1(s^{t+1}) - V_0(s^{t+1})]
\end{aligned} \tag{55}$$

The marginal MP firm's cutoff conditions 35 and 36 can be rewritten (using $m^F$ as last period MP status) as,

$$\begin{aligned}
0 = & \frac{1-\theta(1-\alpha)}{\theta(1-\alpha)} P(s^t) W(s^t) K_{m^F}(s^{t-1})^{1-\nu} \exp\left([Z(s^t) + \eta_{m^F}(s^t)](1-\nu)/\alpha\right) \\
& \times \left\{H_h(s^t)\left[1 + G(s^t)\right]^\nu - H^\star_{hx}(s^t)\right\} - \frac{q(s^t)P(s^t)W^\star(s^t)}{\exp(Z^\star(s^t))} \left[\frac{F^F_1 + (1-m^F)F^F_0}{\exp(\zeta(Z(s^t) - Z^\star(s^t)))} - F^X\right] \\
& - P(s^t)\left[K_1(s^t) - K_0(s^t)\right] + \sum_{s^{t+1}} Q(s^{t+1}|s^t)\left[V_1(s^{t+1}) - V_0(s^{t+1})\right]
\end{aligned} \tag{56}$$

Where $V_1(s^{t+1}) - V_0(s^{t+1})$ is given by 55.

*Capital FOC:* And finally, firms choose capital subject to the following first order condition derived by differentiating value function 9 with respect to capital separately for current non-MP ($m^{F'} = 0$) and MP ($m^{F'} = 1$) firms:

$$1 = \sum_{s^{t+1}} Q(s^{t+1}|s^t) \frac{P(s^{t+1})}{P(s^t)} \left[ \frac{\alpha}{1-\alpha} \frac{W(s^{t+1}) L_{m^{F'}}(s^{t+1})}{K_{m^{F'}}(s^t)} + (1-\delta_k) \right] \tag{57}$$

Where $L_{m^{F'}}(s^{t+1}) = L^D_{m^{F'}}(s^t) + \frac{q(s^t) W^\star(s^t)}{W(s^t)} L^{D\star}_{m^{F'}}(s^t)$ is the sum of average labor demands in 41 and 42.

**Additional Variables**

*Real domestic production, exports and imports*:

$$Y_d(s^t) = \int_i \frac{P_h(i,s^t) y_h(i,s^t)}{P_h(s^t)} = \delta_h^{\frac{1}{1-\rho}} \left( \frac{P_h(s^t)}{P(s^t)} \right)^{\frac{1}{\rho-1}} D(s^t) \tag{58}$$

$$Y_{ex}(s^t) = \int_{i \in \xi^X(s^t)} \frac{e(s^t) P^\star_{xf}(i,s^t) y^\star_{xf}(i,s^t)}{e(s^t) P^\star_{xf}(s^t)} = \delta_x^{\frac{1}{1-\rho}} \left( \frac{P^\star_{xf}(s^t)}{P^\star(s^t)} \right)^{\frac{1}{\rho-1}} D^\star(s^t) \tag{59}$$

$$Y_{im}(s^t) = \int_{i \in \xi^{X\star}(s^t)} \frac{P_{xf}(i,s^t) y_{xf}(i,s^t)}{P_{xf}(s^t)} = \delta_x^{\frac{1}{1-\rho}} \left( \frac{P_{xf}(s^t)}{P(s^t)} \right)^{\frac{1}{\rho-1}} D(s^t) \tag{60}$$

*Gross Domestic Product*

I compute real gross domestic product (GDP) similar to Alessandria and Choi [2007]. Nominal GDP is is the sum of sales of all goods produced domestically.

$$P_G(s^t) Y(s^t) = \int_0^1 [P_h(i,s^t) y_h(i,s^t) + e(s^t) P^\star_{xf}(i,s^t) Y_{ex}(i,s^t)] di + \int_{i \in \xi^{F\star}(s^t)} P_{mf}(i,s^t) y_{mf}(i,s^t) di \tag{61}$$

where the GDP deflator $P_G(s^t)$ is is an aggregation of domestic, export, and foreign MP price indexes,

$$P_G(s^t) = \chi_d(x^t) P_h(s^t) + \chi_x(x^t) P^\star_{xf}(s^t) + [1 - \chi_d(x^t) - \chi_x(x^t)] P_{mf}(s^t) \tag{62}$$

The weights on domestic and export price indexes are their respective shares in GDP: $\chi_d(s^t) = \frac{P_h(s^t) y_h(s^t)}{P_G(s^t) Y(s^t)}$ and $\chi_x(s^t) = \frac{e(s^t) P^\star_{xf}(s^t) Y_{ex}(s^t)}{P_G(s^t) Y(s^t)}$.