Duration & Portfolio Immunization[1]

A bond portfolio is said to be immunized if it is not very much affected by the adverse changes in interest rates. Immunization is accomplished by calculating the duration of the future outflows and then investing in a portfolio of bonds that has an identical duration. To do so, we must understand the concept of duration of a portfolio. Duration of a portfolio of bonds is the weighted average of the duration of the individual bonds in the portfolio. For example, let durations of two bonds be 4 years and 8 years. Assume that $1/4^{th}$ the fund is invested in the bond with 4 years duration and the remaining in the second bond. Then duration of the portfolio is given as 1/4 X 4 + 3/4 X 8, which equals 7 years.

Consider a situation where a portfolio manager has only one cash outflow to make from her portfolio - an amount equal to Rs.10,00,000 to be paid after two years. As there is only one cash outflow its duration is 2 years. Let the portfolio manager have two choices - a bond that matures after one year and the other that matures after 3 years.

The manager can invest all her funds in 1- year bond. She can invest the proceeds of the bond received after one year in a bond that has 1- year maturity. In doing so, she faces reinvestment-rate risk. If interest rates were to decline after 1 year, the funds realised from the first bond have to be reinvested at lower rates of interest.

Alternately, the funds manager can invest in 3-year bonds and sell them at market price after 2 years. The manager in this case faces the interest rate risk. If interest rates were to go up at the time of selling the bond, its price would fall.

Both the strategies - investing in 1-year bond and 3-year bond - entail risks - either reinvestment-rate risk or interest-rate risk. Whereas reinvestment-rate risk occurs when interest rates fall, interest-rate risk occurs when interest rates rise. As the two risks are caused by interest rate movements in opposite directions, it would be possible to construct a portfolio of these two bonds in such a way that the affects are nullified. This process is called immunization. It is achieved by constructing the portfolio in such a way that its duration is equal to 2 years.

Let cash flows for 1-year and 3-year bonds be as follows. There is a single cash inflow of Rs.1070 at the end of one year in case of 1-year bond. In case of 3-year bond, there are annual payments of Rs.80 for 3 years and a payment of Rs.1000 at the end of 3 years. Duration of 1-year bond, which is like a zero coupon bond, is one year. Duration of the 3-year bond can be calculated to be 2.78 years.

We have to now find the proportion of investment to be made in these two bonds. If W1 and W2 are proportions invested in 1-year and 3-year bonds, then

$$W1 + W2 = 1$$

$$W1 \times 1 + W2 \times 2.78 = 2$$

The first equation states that sum of weights is 1. The second equation stipulates that the duration of the portfolio is 2 years. We can get the percentage investments in the two bonds by solving the above two equations. From the first equation we can get W1 to be equal to 1 - W2. Substituting this value of W1 in the second equation we get

$$1 - W2 + 2.78 \times W2 = 2$$

$$1.78 \ W2 = 1$$

$$W2 = 1 / 1.78 = 0.5618$$

Substituting this value of W2 in first equation, we get

$$W1 = 1 - 0.5618 = 0.4382$$

For immunization, the portfolio manager has to invest 56.18% in 3-year bond and 43.82% in 1-year bond. The amount to be invested today to get a cash flow of Rs.10,00,000 after 2 years, assuming a YTM of 10% is given as $10,00,000 / 1.1^2$, which would be Rs.8,26,446. Using the percentages worked out above one can calculate the investments into 1-year and 3-year bonds to be Rs.3,62,149 (8,26,446 X 0.4382) and Rs.4,64,297 (8,26,446 X 0.5618) respectively.

At 10% YTM, the prices of 1-year and 3-year bonds can be worked out to be Rs.972.73 and Rs.950.25. At these prices, the number of 1-year and 3-year bonds

purchased would be 372 (3,62,149 / 972.73) and 489 (4,64,297 / 950.25). Table below explains how immunization is accomplished.

Table : Aggregate Portfolio Value Under Different Yield Assumptions

	YTM at the end of one year (y)		
	9%	10%	11%
Value of 372 1-year bonds at the end of 2 years Value at the end of 2 years after reinvesting proceeds of 1-year bonds at the end of one year - 1,070 X 372 X (1 + y)	4,33,864	4.37,844	4,41,824
Value of 489 3-year bonds at the end of 2 years Value from reinvesting coupons received at the end of first year (80 X 489 X (1 + y))	42,641	43,032	43,423
Value of coupons received at the end of 2 years (80 X 489)	39,120	39,120	39,120
Sale proceeds at the end of 2 years (1,080 X 489 / (1 + y)	4,84,514	4,80,109	4,75,784
Aggregate portfolio value at the end of 2 years	10,00,138	10,00,105	10,00,151

When yield increases, then the portfolio's losses owing to the selling of the 3-year bonds at a discount after 2 years would be offset by the gains from reinvesting the maturing 1-year bonds and first year coupons of the 3-year bonds at the higher rate. Similarly when the yield falls, then the loss from being able to reinvest the 1-year bonds and first year coupons on the 3-year bonds at a lower rate will be exactly offset by being able to sell the 3-year bonds after two years at a premium. Thus the portfolio is immunized from the effect of movements in interest rates in the future.