Why do discriminatory, why have PDs? why only weekly?: An empirical analysis of Indian treasury bonds

Seher Gupta and Rohit Lamba*

July 2016

Abstract

This is short description of a research project in progress on understanding (through a model and data) the key features of the primary market for short-term sovereign debt popularly called Treasury Bills. The data (from the Indian treasury auctions) comprises of all the bids submitted in pay as bid auctions conducted every week by the RBI from Mar 2009-Feb 2011. Bidders are classified into primary and non-primary dealers, the former being the major bidders and winners of notified amount. After laying the field through a series of summary statistics, we present and structurally estimate a multi-unit share auction model, and use it to ask three counterfactuals. First, how would bids and revenue change under a uniform payment rule – the uniform auction? Second, why do primary dealers exist, and what happens if we remove this restriction? And, third, why do we use a multi-unit weekly format, what if it is changed to single unit auction with a higher frequency?

1 Introduction

The idea of the sovereign raising capital to finance its activities has existed almost since the inception of large organized societies – the Mauryan period in India and the Roman empire in Europe are the earliest documented instances. Auctions too have existed since time immemorial. Krishna [2010] reports evidence of the first auctions as far back as Babylon in 500 B.C. It was a matter of time before modern economics figured out the obvious match: to issue sovereign debt through auctions!

There are three broad reasons for auctioning government bonds, as opposed to selling it through any other mechanism be it posted prices, beauty contests or random lotteries. First and foremost – transparency and credibility. Raising debt is a repeated contract where credibility of the state of finances and the ability to repay are paramount. By having a transparent method of allocating debt the sovereign maintains the confidence of the market and the sanctity of its debt.

Second, it is important to foster competition and distributional efficiency. There is something to be said about making access to domestic debt broadly democratic, that is, anyone with

*seher.gupta@nyu.edu, rlamba@psu.edu. This write-up is based on generous discussions with Alessandro Lizzeri and N. Prabhala.

the ability to finance it should be allowed to buy it, with perhaps a requisite insurance structure in place. This helps sustain liquidity, minimize distortions through potential market power, and a maintain steady demand.

Third, and perhaps most important is price discovery. What is the right price of the Indian bond? Who decides that and how? There is a definite understanding following the works of Sandy Grossman amongst others, that prices for securities are best determined by the "market" by aggregating information. A treasury auction lets market participants express their willingness to pay under the watchful eyes of a regulator.

In terms of measurable parameters, a design should achieve the right yield, liquidity and maturity basket for sovereign debt. By getting the right market clearing price, an auction achieves the first target. By encouraging wide and transparent participation, it takes care of the second. And, by making total quantity to be sold an easy instrument, it partakes in the third.

Given that auctions indeed seem like an economically sensible way to issue sovereign debt, the next obvious question to ask is – what is the best auction design to achieve the said objectives. This is a question that has attracted much attention from academics and policymakers alike. Given the paucity of theoretical results, there are no easy answers. A burgeoning literature on structural estimation of multi-unit auctions tries to get to the heart of this fascinating problem.

Typically three key characteristics underly auction design for government securities world over. The auction is always multiunit and typically conducted weekly. It is either a discriminatory (pay-as-bid) or uniform price auction. And, finally, the auction is insured through a set of special bidders called primary dealers who together must bid for the total quantity up for sale.

In this research work in progress we evaluate the heritabilities of each of these design features. While the discriminatory versus uniform question has been debated before (with no conclusive answers), the other two are almost axiomatic in their adherence. It is our endeavor to unpack each of the three and put them up for scrutiny using time series of bid level data from the Indian T-Bill market.

2 Institutional Design

Three types of Treasury Bills (T-Bills) are issued by the Government of India classified by their maturity date from the date of auction- 91, 182 and 364 days. The 91 days auction is held weekly and the 182 and 364 days auctions are held bi-weekly on alternative weeks. The auction calendar including the total amount put up is announced well in advance. While government securities or "G-Secs" are meant for long term investment, T-Bills are money market instruments which "provide investment avenues for short term tenor. By definition, money market is for a maximum of up to one year". They are used for funding transactions in other markets, and meeting short term liquidity requirements.

T-Bills are zero coupon securities, issued at a discount and redeemed at face value. For one piece of paper at a face value of Rs. 100, one will typically see a market clearing price in the set $[95, 99]$. For example, for the auctions conducted between March 2010-February 2012, the

Figure 1: Face Value and Market Clearing Prices

market clearing prices ranged from 97.79 to 99.02, as can be seen in Fig 1. An auction of Rs. 200 crore bond will involve selling of 2 crore pieces of "paper" (will be denoted by $Q = 2$ crore) each worth Rs. 100.[1] The bidders can submit multiple bids in the form of price quantity pairs. A typical bid of the form $\{(p_1, q_1), (p_2, q_2)\}$ with $100 \geq p_1 \geq p_2$ means that the bidder is willing to buy q_1 pieces of paper at a total price $p_1 q_1$, and $q_1 + q_2$ pieces of paper at price $p_1 q_1 + p_2 q_2$, and so on. If both bids win, the bidder makes a transfer of $p_1 q_1 + p_2 q_2$ to the central bank, and is paid $100(q_1 + q_2)$ at the time of maturity.

The main features of the T-Bills auctions are as follows. First, treasury auctions world over are share auctions – multi unit divisible goods auctions. For a total notified amount Q, each player can bid for any fraction of Q. Multiple bidders can "win" the auction. Second, the auction is characterized by a market clearing price. After all the bids are in they are arranged according to descending order of prices. As we go down the list, the price at which the cumulative quantity demanded exceeds Q is christened the market clearing price. Third, is the payment rule. For the quantities won, do the bidders pay the price they bid or the market clearing price? In the example above, if $p^m \leq p_2 \leq p_1$ is the market clearing price, does the bidder pay $p_1 q_1 + p_2 q_2$ or $p^m(q_1 + q_2)$? The former is called discriminatory auction and latter uniform price auction. In India, the T-Bill auction is discriminatory.

Bidders are categorized into primary dealers (PDs) and non-primary dealers (non-PDs). The former is a set of 21 financial institutions that commit to appearing in every auction. They are a group of financial firms or banks that play the role of intermediaries in the G-Sec and T-Bills market. In the T-Bill auctions, the PDs have the following two obligations. First, they agree to bid for a certain minimum fraction of the total notified amount. For example, if all the PDs were to bid at least 5 percent of the total amount, we would have a minimum of 105

[1] 1 crore $= 10^7$.

Bank PD*	Stand Alone PDs**
Bank of America	ICICI Securities Primary Dealership Limited
Bank of Baroda	Morgan Stanley India Primary Dealer Pvt. Ltd.
Canara Bank	Nomura Fixed Income Securities Pvt. Ltd.
Citibank N.A.	PNB Gilts Ltd.
Corporation Bank	SBI DFHI Ltd.
HDFC Bank Ltd.	STCI Primary Dealer Limited
HSBC	Goldman Sachs (India) Capital Markets Pvt. Ltd.
J P Morgan Chase Bank N.A.	
Kotak Mahindra Bank Ltd.	
Standard Chartered Bank	
Axis Bank Ltd.	
IDBI Bank Limited	
Deutsche Bank AG	
Yes Bank Limited	

Note: *Bank PDs are those which take up the PD business departmentally as part of the bank itself.
**Stand alone PDs are Non Banking Financial Companies that exclusively take up the PD business.
Source: Government Securities Market: A Primer, IDMD, RBI, June 2015.

Figure 2: List of Primary Dealers

percent of notified amount bid for even if no non-PDs showed up. Typically, PDs can have different minimal commitments that add up to a number higher than 100 percent ensuring full insurance for the auction. Second, in order to ensure "good prices", the RBI also demands that the PDs win at least 40 percent of their minimal commitments in any given financial year. If a PD fails to meet these obligations, the RBI may take appropriate penal action against the PD. As we will later see in the data, the first can be a binding constraint, second not so much. Figure 2 lists all the PDs.

3 Descriptive Statistics

We have data on T-Bill auctions in two financial years: 2010-11 and 2011-12. For this note we will concentrate on the 91 days bond – a total of 104 auctions. Figure 3 presents the summary stats of the auctions for the two financial years.

The notified amount has a fair amount of variation. The number of non-PDs appearing in the auctions changes quite a bit too. The number of steps is on the smaller side. Given that there are no restrictions on the number of steps a median and mean number of 2 steps are noteworthy.

Two keys statistics often used in the literature are worth defining. Quantity weighted bid is the average price for the total quantity bid. For example for a two step bid $\{(p_1, q_1), (p_2, q_2)\}$, the quantity weighted bid is givem by $p^w = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2}$. Next, the bid cover ratio equals the total quantity bid for in the auction (across all bidders) divided by the notified amount. A high bid cover ratio is considered good for it indicates a high demand for the security.[2]

In financial parlance a bond is characterized by its yield. The yield of a T-Bill is the imputed interest rate due to the difference between the purchase price and face value, adjusted for the

[2]Note that insurance through PDs ensures a lower bound bid cover ratio in excess of 1.

FY2	Mean	Median	Min	Max	SD
Q[1]	4434.29	4000	2000	7000	1925.52
Cut-off Rate[2]	98.50	98.46	98.21	99.02	0.26
# non-PDs	11.67	12	5	20	3.64
# Bidders[3]	32.29	32	26	41	3.58
# Steps Submitted	2.55	2	1	10	1.37
QwBids[4]	98.47	98.41	98	99.03	0.27
Bid-to-Cover Ratio[5]	2.88	2.75	1.63	4.76	0.76

Note:
1. Q is the Notified Amount in Rs. Crores
2. Cut-off Rate is the market clearing price in each auction.
3. Total number of Bidders. This is the sum of number of Primary Dealers (PDs) and those bidders that are not PDs.
4. QwBids are Prices in the bids weighted by the quantities
5. Bid-to-Cover Ratio is defined at the ratio of Total Demand to Total Supply.

FY3	Mean	Median	Min	Max	SD
Q[1]	6563.41	7000	4000	9000	1686.76
Cut-off Rate[2]	97.94	97.94	97.79	98.25	0.10
# non-PDs	18.04	18	9	29	4.34
# Bidders[3]	38.97	39	30	50	4.40
# Steps Submitted	2.19	2	1	8	1.13
QwBids[4]	97.93	97.94	97.70	98.27	0.10
Bid-to-Cover Ratio[5]	2.87	2.73	1.79	5.16	0.77

Note:
1. Q is the Notified Amount in Rs. Crores
2. Cut-off Rate is the market clearing price in each auction.
3. Total number of Bidders. This is the sum of number of Primary Dealers (PDs) and those bidders that are not PDs.
4. QwBids are Prices in the bids weighted by the quantities
5. Bid-to-Cover Ratio is defined at the ratio of Total Demand to Total Supply.

Figure 3: Summary Statistics

number of days to maturity.[3] The time series of auction yields are graphed in Figure 4. In the exact inverse of market clearing prices, the market clearing yields have an increasing time trends. The most important fact to notice is that the market clearing yields lie strictly above the yields at closing time the day before the auction in the secondary market. The difference can be regarded as the markup or the premium of the intermediary.

Next, we delve a bit deeper into the difference between the bidding behavior of PDs and non-PDs. For starters, Figure 5 indicates that PDs bid in more "steps" than the non-PDs. The distribution of steps for PDs first-order stochastically dominates that for non-PDs.

Figure 6 reports the distribution of the fraction of total quantity bid by PDs and non-PDs in any given auction for the whole financial year.[4] PDs first order stochastically dominate non-PDs. Thus, a PD is more likely to demand a higher proportion of the notified amount that a non-PD.

Given that PDs have a higher demand, it is instructive to explore how they fare in terms of prices. Quantity weighted price defined above records the average price bid for the quantity

[3] The yield of a T-Bill is computed as:

$$\text{Yield} = \left(\frac{100 - P}{P}\right) \times \left(\frac{365}{D}\right) \times 100$$

where P is the purchase price, and D is the number of days to maturity.

[4] Figure 6 plots the empirical cumulative distribution function of the total quantity demanded by a PD or a non-PD in an auction.

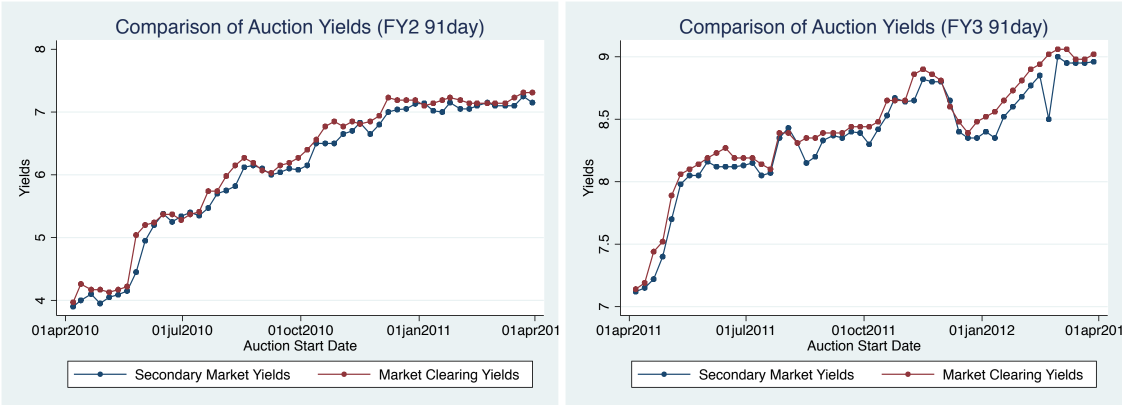


Figure 4: Yields in the Auctions and Secondary Market

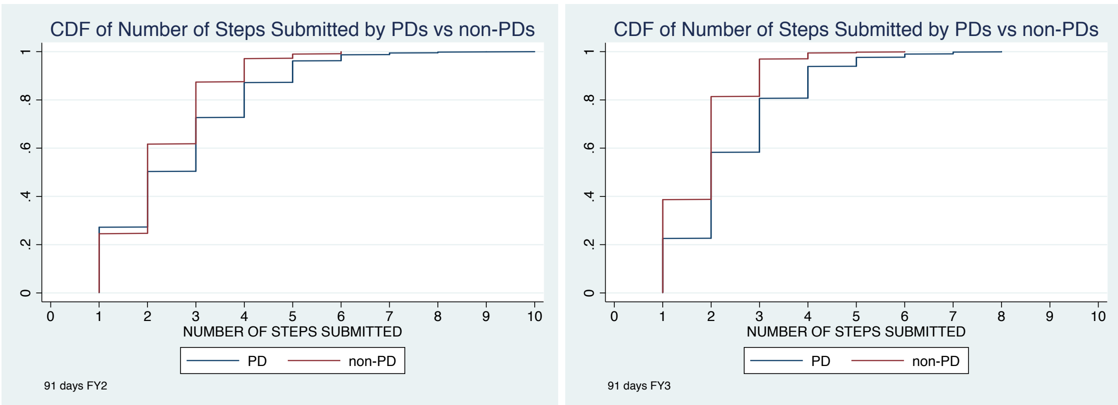


Figure 5: Distribution of the number of steps for PDs and non-PDs

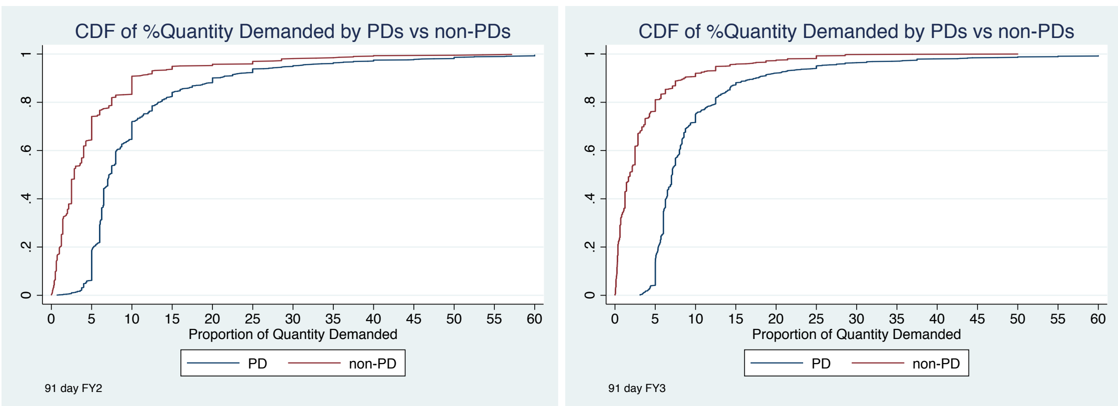


Figure 6: Distribution of quantities demanded by PDs and non-PDs

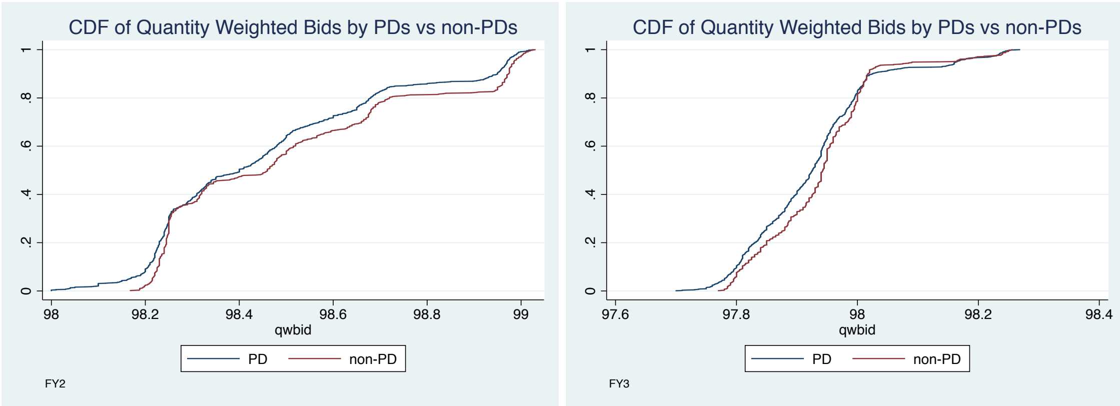


Figure 7: Distribution of quantities weighted prices bid by PDs and non-PDs

demanded. It gives a measure of the average willingness of to pay because it is the per-unit price a bidder will pay if all her bids are accepted. On this metric the statistical relationship is reversed. The distribution for non-PDs first-order stochastically dominates PDs. Figure 7 documents this. Put together Figures 6 and 7 imply that at every percentile PDs bid for higher quantities but lower prices than non-PDs.

Since lower prices translates to higher yields, Fig 7 must imply that PDs bid at higher yields. Furthermore, how do the PDs and non-PDs differ in the premiums they demand on the yields in auction for a given secondary market price? Figure 8 plots the bid yields[5] against the secondary market yields at closing time the previous day. It is clear that: (1) PDs ask for a higher yield than the non-PDs, and (2) they charge a higher premium than the secondary market yields.

In terms of numbers, Fig 9(a) reports that on average a PD bids at an yield that is 178 basis points higher than a non-PD in the financial year 2010-11 (FY2). This difference is 52 basis points in 2011-12 (FY3). While a PD demands 10% of the total supply on average, a non-PD demands at most half of that on average. We can conclude that primary dealers ask for larger quantities at higher yields. Not surprisingly then, the last column of Fig 9(a) shows that PDs on average win much lower proportions of their demands. Fig 9(b) tells us that a PD asks for a yield premium that is 113 (29) basis points higher than that of a non-PD in FY2 (FY3).

Finally, we plot the entire time series of quantities demanded for a select few PDs and non-PDs (four each) for all the auctions. Figure 10 plots the quantity bid and quantity won by four different PDs. It can be immediately noted that for bidder (b) and (d) the minimum criterion set by RBI binds often. Also, for them the quantity won is highly correlated with its decision to spike the quantity it bids above the minimum level.[6] For bidder (a) the quantity bid is higher than its minimum level, but quantity won is not highly correlated with bid spikes. Bidder (c) often departs from its minimum level and often wins when he does so.

Figure 11 plots the participation decision, quantity bid, and quantity won by four non-PDs. Bidder (b) and (c), a Nationalized Bank and a Private Bank respectively, are more regular than

[5]Bid Yield is the quantity-weighted yield at which a bidder demands the total quantity in a bid. It is computed as the yield corresponding to the quantity-weighted price of a bid.

[6]Note that quantity won is an endogenous object. It depends on the price component of the bid, and the bids of all other bidders through the market clearing price.

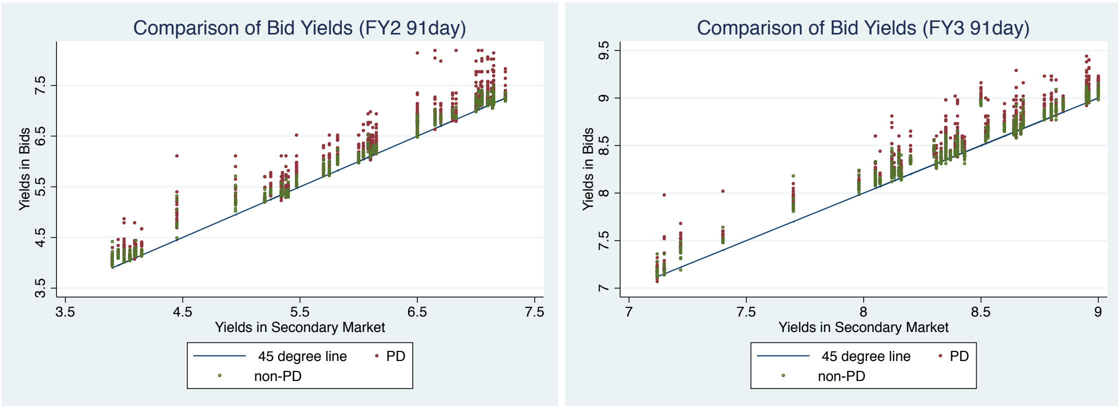


Figure 8: Difference in yields between by PDs and non-PDs in comparison to the secondary market

	Bid Yield		Bid Quantity		% Demand Won	
	PD	Non-PD	PD	Non-PD	PD	Non-PD
FY2	6.282	6.104	10.94% (7.25%)	5.31% (2.86%)	28.58% (12.51%)	47.08% (44.55%)
FY3	8.492	8.440	10.14% (7.11%)	3.62% (1.86%)	29.85% (19.80%)	52.72% (56.90%)

Note: 1. Bid Yield refers to quantity-weighted bid-yield averaged across auctions.
2. Bid Quantity is the % of total Notified Amount demanded by a bidder. Median is in the parenthesis.
3. % Demand Won refers to the percentage of own demand won by the bidder. Median is in the parenthesis.

(a) Bidding Behaviour of PDs and non-PDs

	PD		Non PD	
	Mean	Median	Mean	Median
FY2	0.239	0.19	0.126	0.11
FY3	0.149	0.12	0.086	0.07

Note: Normalized Bids are defined as quantity-weighted yields minus the closing yield in the secondary market on the day before the day of the auction.

(b) Analysis of Normalized Bids

Figure 9: Summary Statistics of Bids

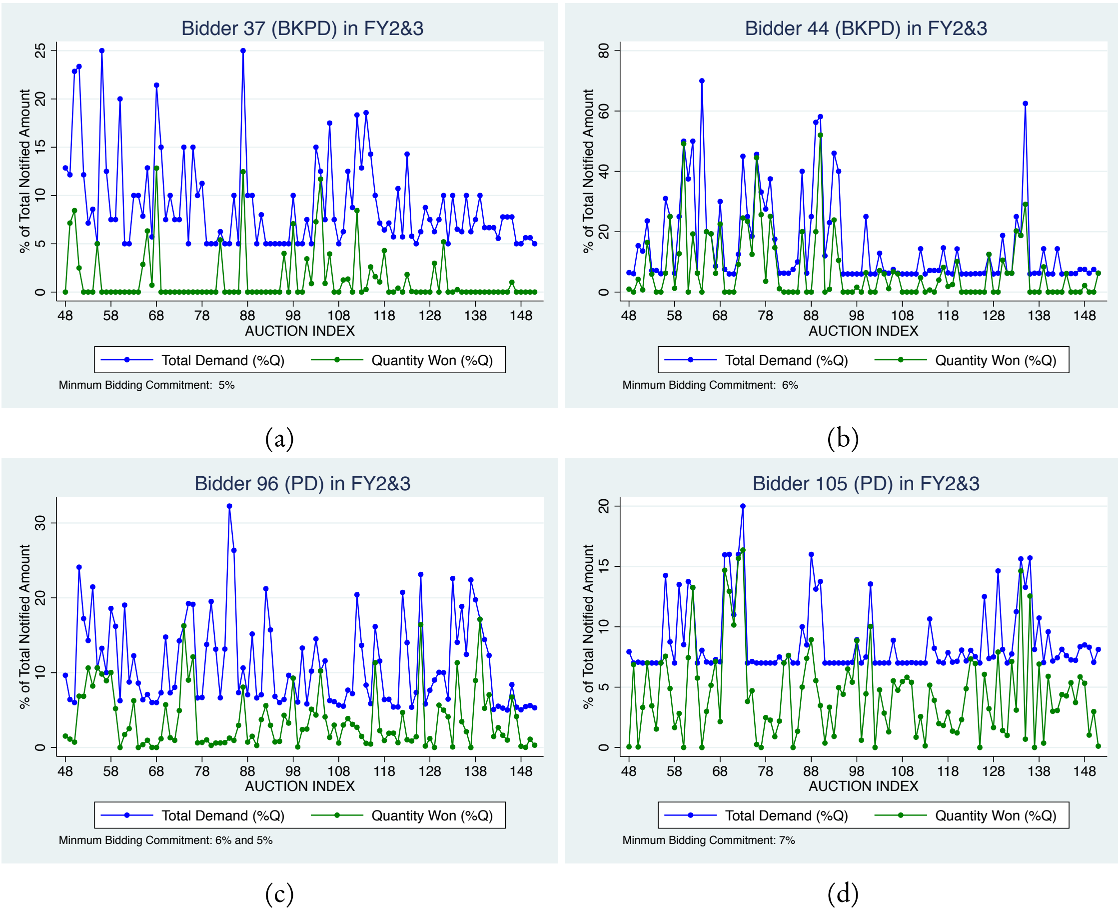
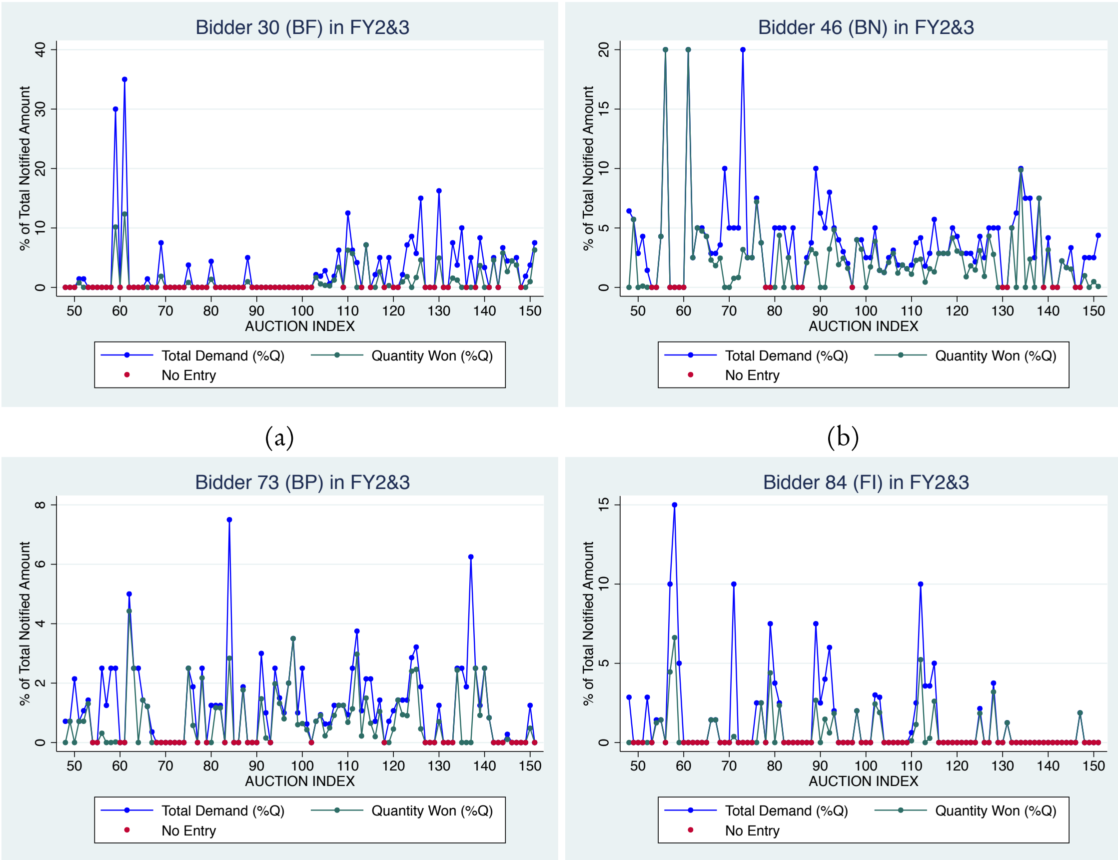


Figure 10: Bidding behavior of PDs for all auctions



(a) (b) (c) (d)

Figure 11: Participation and Bidding behavior of non-PDs for all auctions

the Foreign Bank (bidder (a)) and Financial Institution (bidder (d)). Conditional on participation, the non-PDs win a strictly positive quantity more often than the PDs.

4 Why structural estimation

Why do we want to structurally estimate the auction? There are two key reasons. First, we want to find out the marginal valuations of the bidders. This is a primitive to any welfare and policy analysis of the auction. Putting structure on behavior and backing out marginal valuations allows us to calculate bidder surplus and evaluate the efficiency of the auction.

Second, we want to find out the best possible mechanism of allocating sovereign debt, where "best" is defined by the triple criterion of yield, liquidity and maturity. Currently the Reserve Bank of India issues T-Bills through a weekly held multi unit discriminatory auction with primary dealers guaranteeing a minimum bid cover ratio of one. In order to ask if this is the best possible allocation mechanism, we must evaluate counterfactuals. By structurally estimating the model, we examine three critical features of the auction design: that it is discriminatory, that it demands obligations through PDs, and that it is weekly share auction.

5 Counterfactuals

Since Milton Friedman advocated the use of uniform price auction to sell treasury securities a vibrant academic literature has sprung up around the question of multi unit auction design with immediate policy relevance. It may be tempting to think that the discriminatory auction gives a smaller premium to the bidders and higher revenue to the auctioneer, but that simple reasoning will miss the important fact of the market clearing price is endogenous to the payment being uniform or discriminatory. Friedman argued that uniform price auction encourages greater and more aggressive participation. In the absence of a theoretical ranking, the question remains quite open as empirical studies have proven inconclusive.

World over central banks assign a set of bidders to be primary dealers who agree to certain commitments in order to ensure the success of the auction in form of the entire notified amount being sold. Basically it is an insurance mechanism for an "auction failure" may signal a lack of confidence in the sovereign's debt. We ask what are the costs and benefits of employing PDs. From Figure 7 we know that non-PDs are willing to pay a higher price than the PDs for the average quantity they bid. Given that PDs win a larger fraction of quantities, this points to clear cost associated with intuition of PDs. On other hand not having PDs might, with some probability, lead to an an ex post realisation of a bid cover ratio of less than 1, which would mean a failed auction. What is the probability of such an event and its associated costs?

In perhaps the first clear modelling of multi unit "share auctions" Wilson [1979] showed that under some conditions allocating the whole good to one bidder in a single unit auction generates higher expected revenue for the seller than allowing for multiple bidders to win shares of the auction. There are many good reasons for issuing short-term debt through weekly multi-unit auctions – liquidity and competition being two. Can these objectives be left unharmed by issuing debt through a series of single unit auctions at a higher frequency?

6 Models under consideration

In order to structurally estimate the model and test for counterfactuals, we need a theory. The existing theory is sparse but handy. The two leading estimable multi unit auction auctions are Wilson [1979], and Hortaçsu and Kastl [2012]. We present the basic model below and follow it up with the main limitations.

6.1 Key Ingredients

- T auctions.
- Q_t identical and indivisible units with N potential bidders.
- A subset of bidders may have zero value and choose not to participate.
- Bidders are symmetric, risk neutral and have independent private values.

- Each player bids for up to K objects based on a weakly decreasing value schedule

$$v_{it} = (v_{it}(1), .., v_{it}(y), .., v_{it}(K))$$

with a well defined density over $\mathcal{R}_+^K$.

- Corresponding demand schedule is defined as

$$d_{it}(p) = \max\{y \mid v_{it}(y) \geq p\}$$

6.2 Auction Rules

- A bid is a (weakly) decreasing price schedule:

$$p_{it}(.) = (p_{it}(1), .., p_{it}(y), .., p_{it}(K)))$$

where $p_{it}(y) \geq p_{it}(y+1)$.

- Equivalently a (weakly) decreasing bid schedule

$$y_{it}(p) = \max\{0 \leq y \leq K \mid p_{it}(y) \leq p\}$$

- A range of consecutive quantities may be bid at the same price. If so, the range of quantities, is called a *step*.

- The **market clearing price** is determined by the point where supply and aggregate demand curves intersect. If this happens at multiple prices (with step functions), we choose the highest such price.

$$p_t^c = \max_p \left\{ p \mid \sum_{i=1}^{N} y_{it}(p) = Q \right\}$$

Because supply curve will intersect the horizontal part of aggregate (step) demand function, with positive probability the excess supply is distributed according to some rationing principle which is implicit in the definition.

- Each bidder pays the unit bid on every unit she wins. For example if there are three units for sale and buyer i bids $(p_{it}(1), p_{it}(2), p_{it}(3))$, and the market clearing price p^c is such that $p_{it}(2) > p^c > p_{it}(3)$, then bidder i gets assigned 2 objects with a payment $p_{it}(1) + p_{it}(2)$.

- In the data though we do not directly observe y_i.[7] We observe price and the "additional" quantity demanded at that price. For example if there 40 units for sale, a typical bid would look like $((50, 10), (40, 10), (30, 10))$ where each of these is a price-quantity pair. From this, we can construct a vector: $(y_i(p_0), .., y_i(30), .., y_i(40), .., y_i(50), .., y_i(p_K))$ with three steps at prices 30, 40 and 50; and $y_i(p) = 30$ for $p \leq 30$, $y_i(p) = 20$ for $30 < p \leq 40$,

[7] Suppressing t for simplicity.

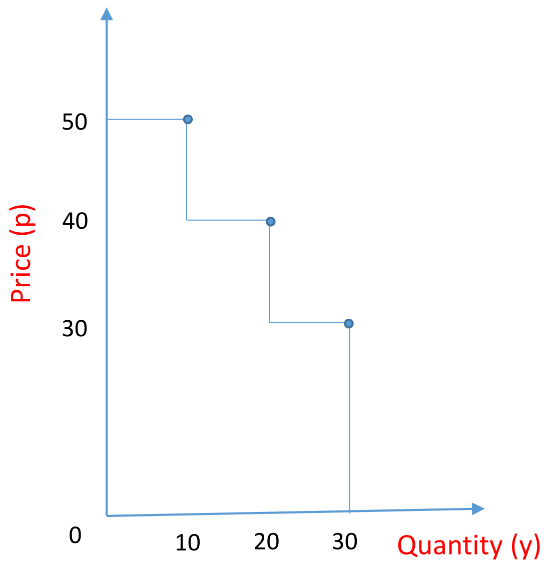


Figure 12: Example of a price quantity bid

$y_i(p) = 10$ for $40 < p \leq 50$ and $y_i(p) = 0$ for $p > 50$ (see Figure 12). If market clearing price is 35, the bidder i gets 20 units at price 90.

- When solving simultaneously for all bidders strategies, we invoke Bayes Nash Equilibrium.

6.3 Residual Supply and Equilibrium

- We assume that bidder valuations and participation decisions are independent across time. The gives us time independent strategies for the agents.

- At price any price p, the residual supply function faced by bidder i is given by:

$$S_t^{-i}(p) = \max\left\{Q_t - \sum_{j \neq i} y_{jt}(p, v_{jt}(.)), 0\right\}$$

  where $y_{jt}(p, v_{jt}(.))$ denotes bidder j's strategy in auction t as a function of her valuation. Analogously, the inverse demand notation for strategies is given by $p_{it}(., v_{it}(.))$.

- At price $p_{it}(y) = p$, bidder i wins at least quantity y as long as the residual supply function is greater than or equal to y. This probability is expressed as:

$$G_{it}(y; p) = \mathbb{P}\left[S_t^{-i}(p) \geq y\right]^8$$

- In each auction t, bidders play symmetric pure strategies $p_t(., v_{it}(.))$ so $G_{it} = G_t \; \forall i$.

[8] Implicit in this definition is an assumption of rationing: sharing rule when the market supply intersects the aggregate demand curve on the horizontal part of a step.

- Interim Expected Payoffs are given by

$$\Pi_t\ (p(.), v_{it}(.)) = \sum_{y=1}^{K} G_t(y; p(y))\ [v_{it}(y) - p(y)] \tag{1}$$

- The equilibrium assumption states

$$\Pi_t\ (p_t(., v_{it}(.)), v_{it}) \geq \Pi_t\ (p(.), v_{it}(.)) \quad \forall\, p(.),\ \forall\, v_{it} \tag{2}$$

6.4 Estimation

Write down the first-order condition.

6.5 Limitations

As described above, in the actual data, the bidders are divided into PDs and non-PDs. Following Hortaçsu and Kastl [2012], we want to separately estimate marginal valuations for these two groups. In addition the non-PDs do not turn up for every auction. So, we want to add a entry decision for non-PDs.

Next, we are not convinced that the existing private values model is the right foundational framework. If the PDs are truly acting purely as intermediaries, there must be a mark-up model in the background that drives the marginal valuations. Moreover, there must be strong common value component to the sovereign bond.

Finally, the current state of the art in estimation strategy does not allow for estimation of marginal valuations for single-step bids. Given that there is a large number of single step bids, we want to come up with a better theoretical model and estimation methodology to take this data characteristic into account.